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THE SUPERMARKET: AN EXAMINATION OF A QUEUING MODEL

by

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ABSTRACT

This study is concerned with the application of a queuing model to the check-out operation in a supermarket. Economical and efficient check-out service is an objective desired by both management and the customer.

The data used in this study was collected through the medium of a survey conducted in an Edmonton supermarket over an eight week period between May 31 and August 14, 1965.

The queuing model employed in this study was one which derived the state probability distribution in the case where the number of arrivals per unit of time was a Poisson variable and service time was distributed by a negative exponential distribution.

The major conclusions that resulted from analysis of the data were:

1. The number of customers arriving at the check-out counters per unit of time, was generated by the Poisson variable. This implied that the length of time between two successive arrivals was distributed by a negative exponential function.

2. The service time distribution for servicing customers was not distributed by a negative exponential distribution. It was determined that an Erlang distribution was a good approximation of the service time distribution.

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INTRODUCTION: THE NATURE OF THE PROBLEM

The subject of this thesis is the investigation of a queuing model which applies one of the techniques of Operations Research to a well-known problem in retailing. More specifically, it involves the examination of a queuing model designed to determine optimal rules for the check-out operation in a supermarket. The model was advanced in a doctoral dissertation by John Yaotung-Lu.[1] He addressed himself to the problem of how to provide a high grade of check-out service in the most economical manner.

Competition in the retailing trade is characterized by the large number of variables available to the consumer. Shopping at a large supermarket is an experience familiar to most of the general public. Each shopper, as he pushes his cart through the store, selects his purchases, and arrives at the check-out counters, cannot help but wonder if there is a way of improving the operation.

Management can do very little to adjust the flow of customers into the store, but it can adjust the number of check-out counters available to the customer.

In most supermarkets the allocation of manpower necessary to provide the check-out service, as well as the control of the number of check-out counters in operation at any one time, is left to the

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[1]John Yaotung-Lu, Use of Queuing Theory in Determining Optimal Supermarket Check-out Rules, (Ann Arbor, Michigan: University Microfilms, Inc., 1959). (A dissertation submitted to the School for Advanced Graduate Studies, Michigan State University, 1959).

discretion of the store manager. In the Edmonton supermarket selected to test Yaotung-Lu's model, decisions regarding the number of check-out counters operating at any one time are based on the manager's experience and a subjective service standard employed by the company.

If management must make a decision regarding the amount of labor and service facility required to meet a given flow of customers, knowledge of the functional relationships which exist between the service facilities, labor, and the customers will advance the decision-making process.

Knowledge of the relationships existing between the check-out operation, the merchandise display area, and the storage area may aid management in making decisions related to store design. While there are many stores that have an adequate relationship at present, there are others that could improve their operations, both for their own profit and for better service to the customer.

The queuing model developed by Yaotung-Lu is chiefly concerned with procedures that can be used to decide the number of check-out counters that should be in operation to handle the customers ready for check-out service at any given time. For example, with four employees available, the decision whether it is better to have four check-out counters operating with a cashier alone or two check-out counters operating with a cashier assisted by a wrapper, may be undertaken with greater insight.

Finally, based on the queuing model advanced by Yaotung-Lu, the company may formulate new service policies and/or ratify the

existing service policy. That is, the amount of service allowed by the company may be altered because the management has developed new thinking with respect to the benefits the company may receive from the operation.

Hence, the operational problem involved in the check-out operation of a supermarket may provide management with a wider range of decision making tools when it is evaluated in terms of a queuing model.

1: WHAT IS A QUEUING PROBLEM?

A waiting time problem arises when either units units requiring service or the facilities available for providing service stand idle.[2] Depending on their structure, problems involving waiting time fall into two categories. The first type of problem, the "waiting line" problem, involves arrivals which are randomly spaced and/or service time of random duration. This class of problem includes situations which require the determination of the optimum number of service facilities and/or the optimum arrival rate.[3]

Secondly, the "sequencing" type of waiting problem is neither concerned with controlling the times of arrivals nor the

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[2]Eugene M. Grabbe, Simon Ramo and Dean E. Wooldridge, Handbook of Automation, Computation and Control, (New York: John Wiley and Sons, Inc., 1958), p. 15 - 73.

[3]*Ibid.*, p. 15 - 73.

number of service facilities, but rather is concerned with the sequence in which service is provided to available units by a series of service counters.[4]

In the case of the supermarket, the first type of problem is more applicable. There are many operational problems involving the flow of customers[5] in which the following conditions are observed:

1. The service facilities must remain unused, not only because of the lack of customers in quantity, but also because of the nature of the time spacing between customer arrivals.[6]

2. Units requiring service must wait for service because there is a shortage of service facilities.[7]

Either of these conditions results in the formation of a waiting line. In the first instance the units in the waiting line consist of service facilities, and in the latter it would be a queue of input units. The system will nearly always have waiting customers or idle service facilities and/or personnel, with the associated costs.[8] The problem is to minimize the sum of these costs.

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[4] Ibid., p. 15 - 74.

[5] In queuing theory, the term customer is not restricted to a person. It can mean an automobile waiting to enter a toll gate, a telephone call waiting to enter a long distance trunk line, or an aircraft waiting to leave an airport by way of a specific runway.

[6] Yaotung-Lu, op. cit., p. 1.

[7] Ibid., p. 1.

[8] Maurice Sasieni, Arthur Yaspan, and Lawrence Friedman, Operations Research - Methods and Problems,(New York: John Wiley and Sons, Inc., 1959), p. 125.

The shortage or surplus of service facilities is usually brought about by the inability of the system to decide in advance the right amount of service facilities to provide for its customers. This inability, in turn, is explained by the random elements which influence demand for service in time and quantity. If the laws governing arrivals, servicing times, and the order in which arriving units are taken into service are known, then the nature of the waiting situation can be studied and analyzed mathematically.

Under these conditions a queuing problem arises. The problem is to change the behavior of the arriving units and/or the service facilities in order that the queuing process may be operated as efficiently and economically as possible. In the case where the queue consists of input units, it may be assumed that the total variable cost (TVC) of operating a system that involves a queuing process consists of waiting time cost (WTC) and operating cost (OC).[9]

In the case of Figure 1, total variable cost using (x) facilities, is the sum of the operating cost (xa), and the waiting time cost (xb). Yaotung-Lu defined the waiting time cost as the cost of losing a potential customer because of insufficient service facilities. He felt that the waiting time cost for units being serviced decreases at a decreasing rate as more service facilities are added to the system. He based this assumption on the fact that waiting time cost is a joint function of the behavior of the arriving units and service facilities.

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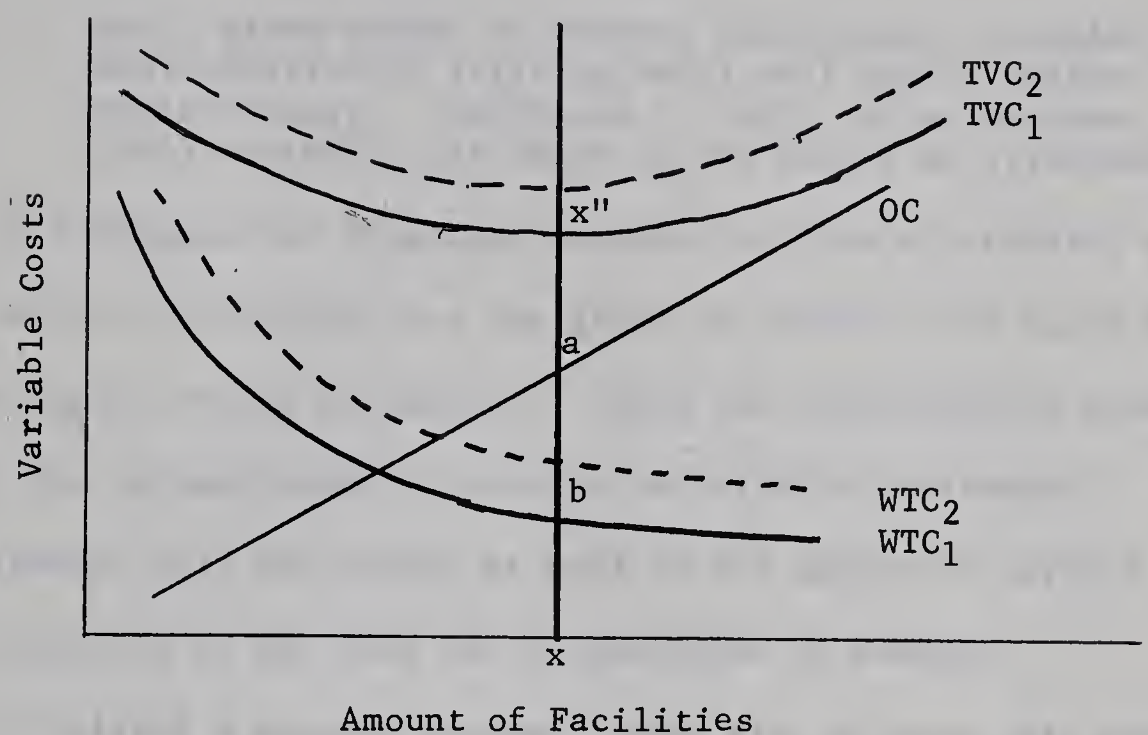
[9] Yaotung-Lu, op. cit., p. 2.

Figure 1. Total variable cost of operating a queuing process.[10]



As indicated by the dotted curves in Figure 1, a new waiting time cost function and a corresponding total variable cost function can be drawn for a change in the behavior of the arriving units. On the other hand, operating cost may be assumed to increase in proportion to the amount of facilities provided.

There are at least three ways to minimize the total variable costs of the system:[11]

1. For a given behavior of arriving units, determine what is the right amount of service facilities which corresponds to the minimum point on the total variable cost curve.

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[10]*Ibid.*, p. 2.

[11]*Ibid.*, p. 3.

2. The minimum point along the lowest total variable cost curve can be found by varying the two variables - behavior of arriving units and service facilities.

3. For a given amount of service facilities, determine what behavior of arriving units will give a minimum variable cost. In Figure 1, (xx") is the minimum total variable cost based on the curves as illustrated.

To determine the relations between the flow of arriving units, amount of service facilities, and the grade of service, the tools of probability theory should be applied. Once the relationships have been determined, the optimum grade of service can also be determined. The flow of customers into the system as well as the amount of service facilities required at any time can be specified in advance.

In solving a queuing problem, there are two ways that probability theory can be applied: the simulated sampling approach, and the mathematical approach.

In the simulated sampling approach, a procedure known as the Monte Carlo method is often used.[12] The Monte Carlo technique uses a table of random numbers and empirically derived probability distributions to determine statistics on arrivals and service time. Thus a study of the effects of changing conditions can be simulated without the researcher waiting for data to be collected over a long period of time. With the rapid development of high speed computers the simulated sampling approach has become more useful and more widely used.

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[12] Ibid., p. 4.

Yaotung-Lu pointed out that the mathematical approach begins with specifications of probability distributions regarding service times and customer arrivals.[13] Based on these specifications and statements about the probability of there being a given number of customers in a waiting line under various conditions, relationships that describe the queuing process can be derived.[14] Yaotung-Lu maintained that these relationships can be solved for such quantities as the expected length of waiting line and the average number of customers in the system.[15] When costs arising from the operation of service units and waiting time are known, the conditions under which a minimum cost is attainable can be derived analytically.

Solution of a problem by a mathematical approach has an orderly structure and a neat appearance. However, sometimes it is difficult, if not impossible, to specify the arrival and service distributions explicitly.[16] Frequently, even if the arrival and service distributions can be represented in terms of probability distributions, a researcher is unable to derive mathematical statements describing the queuing process. If this is the case, the simulated sampling approach may be utilized.

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[13] _Ibid_., p. 4.

[14] _Ibid_., p. 4.

[15] _Ibid_., p. 4.

[16] _Ibid_., p. 4.

The model employed in the study by Yaotung-Lu is based on the mathematical approach. It attempts to minimize total variable cost with respect to the amount of service facilities and to a given behavioral pattern of arriving units.

2: THE SUPERMARKET - A COMMON QUEUING PROBLEM

In the operations research literature, many authors have made reference to the supermarket as an excellent example of a problem to which queuing theory may be applied.[17] They state that the problem in the supermarket is to design the service facility in a manner which will keep the waiting time sufficiently low and thus forestall the leaving of the supermarket by a high percentage of the impatient customers. The operation is single- or multiple-channel, depending on the number of service units and the size of the store. A small store would have only one service unit and would therefore be classed as a single-channel operation.

When one is dealing with a supermarket, it is essential to remember that the store is only a subsystem operating within the larger system-society. For example, the demands upon the service units in

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[17]Sasieni, Yaspan and Friedman, op. cit., p. 152; Paul J. Burke, "The Output of a Queuing System," Journal of The Operations Research Society of America, 1956, 4, p. 699: Philip M. Morse, Queues, Inventories and Maintenance, (New York: John Wiley and Sons, Inc., 1962), p. 3.

the supermarket are independent in that they are made by numerous individuals who act, to a degree, independently of one another.

Yaotung-Lu, when reviewing the supermarket for his study, recognized this fact when he stratified his sample for arrival rates to try to obtain some degree of consistency in his model. He seemed to recognize the peak periods when customer traffic intensified. However, he did not apply the same reasoning when he reviewed the service mechanisms that operate within the supermarket. This point will be further discussed in Chapter II of this dissertation.

3: OBJECTIVES OF THE STUDY

The purpose of the study was to test the queuing model developed by Yaotung-Lu by using data obtained from an Edmonton supermarket. Yaotung-Lu maintained that the check-out operation in a supermarket appeared to have all the necessary characteristics of a typical queuing problem.[18] In the supermarket there are customers demand service. As each customer reaches a service unit, he receives service. After a finite service time, the customer leaves the service unit. If the service unit is not immediately available to him, he must wait his turn; that is, the customer must join a queue. How long the customer will be in the waiting line is dependent on two factors: the number of service units in operation, and the number of customers that preceeded him into the store.

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[18]Yaotung-Lu, op. cit., p. 5.

A second objective was to develop alternative solutions to the overall optimizing problem observed by Yaotung-Lu. Specifically, an analysis was carried out to determine whether the problems encountered by Yaotung-Lu would be encountered under the study conditions in the Edmonton supermarket. When these analyses were performed, alternative methods for solving the problem were proposed.

A final objective was to analyze critically the underlying assumptions of Yaotung-Lu's model in order to determine whether the model had an adequate base for solving the problem posed by the check-out operation in a supermarket.

In order to analyze the model, statistical tests based on hypothetical probability distributions were employed. A regression analysis was employed to further test the model.

4: LIMITATIONS OF STUDY

The queuing model developed by Yaotung-Lu was based on several critical assumptions. The check-out operation of an Edmonton supermarket was used to test the validity of these assumptions.

The first limitation encountered the assumption that the supermarket was a closed system, when in fact it was part of a much larger chain store system. The management felt that the policy of the store was synonomous with the policy of the chain management and that this was not a significant limitation. Management policy has little

to do with the rate at which customers arrive at the store for their weekly shopping. It is the larger system in which the supermarket must operate that has a strong influence on the operation of the supermarket and the congestion problems that face management. This is the real limitation.

A second limitation arose when the sales records were made available on a restricted basis.[19] The average arrival rate was found to be related to the sales per unit of time. That is, the periods with higher traffic intensity were also the periods with the larger sales volumes and a higher dollar volume per order.

Another limitation of the study was the human aspect of the research. The sample data was obtained by one person over a period of three months. This restricted the amount of data obtained. For example, the data for arrival rates and the data for service times were obtained from similar days during the study, but better results might have been obtained if the customers used in the arrival data could also have been used in the service time data. Instead, as previously mentioned, the data is for corresponding periods, but not for corresponding individuals. This limitation was created by the lack of funds.

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[19]Based on the sales records, the week was divided into sample periods so that within each period the assumption of a stable average arrival rate would be more tenable.

5: DEFINITIONS AND SYMBOLOGY

To facilitate a concise treatment of the theoretical and empirical analysis, some shorthand symbols and their definitions are offered here. Other terms and symbols which were used in the study were adapted from standard economic and statistics tests. While some of the symbols are not employed directly, this section offers a glossary of terms which may confront the reader when reviewing queuing models.

Queue. The literal or figurative waiting line which forms before a service or selling unit when the rate of demand on the unit exceeds the unit's ability to process the customers. In short, it is the waiting of customers. The symbol L_q was assigned to the expected length of such a line.

Customer. These are defined as the units in the system upon which service was to be performed.

Server. These are defined as a person or service unit which performs an operation or a series of operations for a customer. The operations have the basic characteristic that they take some time to perform, thus a finite number of customers are serviced in a finite period of time.

Service Units. These are the locations at which the operations on the units requiring service are performed. In some economic textbooks they are called service stations rather than service units.

Queue Discipline. This is defined as the order in which the customers leave a queue for service, or the order in which units are served. In the case of the supermarket queue, the customer was is first-in the queue is the customer who is first-out (FIFO). In other cases, the order in which a customer is selected for service is determined by random selection, priority ranking, or some other technique.

System's Input. The nammer in which units (customers at a counter, cars at a toll gate, planes at an airport, etc.) arrive and become part of a waiting line.[20]

Service Policy. This is defined as the limitation on the number of service units or the amount of service that is rendered, or is allowed by company policy.

System's Output. This is the service provided and its duration. To describe a queue completely, all four factors (system's input, service units, service policy, and the queue discipline) must be clearly defined.

Queuing Times or Waiting Lines. There are the number of time units (seconds, minutes, hours, etc.) which are required by a customer after entering a queue and before a server renders service to the individual.

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[20] C. West Churchman, Russel L. Ackoff, and E. Leonard Arnoff, Introduction to Operations Research, (New York: John Wiley and Sons, Inc., 1961), p. 391.

<u>Arrival</u>. Each customer who arrives at a service unit (demanding service) is referred to as an arrival.

<u>Arrival Rate</u>. This is the average number of arrivals for service who enter a queue waiting for service, or enter the service facility itself[21] in some unit time period. The symbol λ is assigned to represent this rate of flow.

<u>Mean Arrival Rate</u>. This is defined as the expected number of arrivals which occur in a time interval of length unity.[22] That is, the average rate at which customers arrive at the service unit for service.

<u>Service Rate</u>. This is defined as the average ability of a single server or service unit to process the customers in some unit time period.[23] The symbol μ is used to represent this variable.

<u>Mean Servicing Rate</u>. For a particular station, this is the conditional expectation of a number of services completed in one time unit, given that servicing was going on throughout the entire time unit.[24]

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[21]It was assumed that no reneging took place; therefore, the number that entered the queue equalled the number that entered the service unit.

[22]Sasieni, Yaspan and Friedman, <u>op</u>. <u>cit</u>., p. 127.

[23]This was the reciprocal of the average time required for one server to perform the entire operation on one customer.

[24]Sasieni, Yaspan, and Friedman, <u>op</u>. <u>cit</u>., p. 128

6: PLAN OF PRESENTATION

Chapter II is concerned with the model developed by John Yaotung-Lu. It involves the reviewing of his model in study form and the formation of hypotheses to test the model.

Chapter III discussed the general operating characteristics of the supermarket under study.

Chapter IV is devoted to the testing of the hypotheses set forth in Chapter II.

The statistical techniques applied are the following: multiple regression (analysis of variance), the calculation of simple correlation coefficients, the use of the chi-square test for the goodness-of-fit of a theoretical distribution to an empirically derived distribution, and the calculation of the means, variances, and standard deviations of the various distributions under consideration.

Chapter V is designed to compare the results of the study as they would evolve from Yaotung-Lu's model with the results that would evolve from the revised model that resulted with the acceptance of the hypotheses tested in Chapter III.

Chapter VI summarises the results of the study. Some direction for future study is also indicated.

CHAPTER II

A REVIEW OF THE STUDY CONDUCTED BY JOHN YAOTUNG-LU

1: THE QUEUING MODEL ADVANCED IN YAOTUNG-LU'S STUDY

Various queuing models can be constructed by altering the specifications regarding: (1) the queue length, (2) the queue discipline, (3) the number of service channels, and (4) the probability distributions of customer arrivals and service times.[1] Feller discussed the queuing phenonena as a simple time-dependent stochastic process. He derived the state probability distribution in the case where the number of arrivals per unit of time was a function of a Poisson variate and service time was distributed by a negative exponential distribution.[2] Feller presented the derivations of state probability distributions in a highly technical form. He stated that the time interval between two consecutive arrivals had a negative exponential distribution when the generating function of the number of arrivals per unit of time was a Poisson variable. In addition, he formulated the partial differential equation for the generating function of the state probability distributions.

Based on Feller's mathematical statements, Yaotung-Lu developed his queuing model with the intention of optimizing the check-out operation

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[1]Yaotung-Lu, op. cit., p. 7.

[2]W. Feller, An Introduction to Probability Theory and Its Applications, (New York: John Wiley and Sons, Inc., 1950), pp. 363 - 367.

in a supermarket by providing optimal check-out rules. Yaotung-Lu's model includes: (1) multiple exponential channels with Poisson arrivals, (2) a strick queue discipline, and (3) an infinite queue.[3] By multiple exponential channels, Yaotung-Lu meant that there is more than one service point and the time needed for a customer to be served follows the negative exponential distribution. He stated that the Poisson arrivals refer to the assumption that the number of customers arriving per unit of time is characterized by the Poisson variate. The strict queue discipline follows the "first-come first-served" concept. The infinite queue refers to situations in which every arriving customer must join the queue, no matter how long the queue happens to be. Therefore, theoretically, the queue may become infinite.[4]

2: ANALYSIS OF THE ARRIVAL DISTRIBUTION--YAOTUNG-LU

The hypothesis adopted by Yaotung-Lu throughout his study was that the customer arrivals consisted of a Poisson process. Based on Feller's analyses, the number of arrivals per unit time, being a Poisson variable, implied that the time interval between two consecutive arrivals had the negative exponential distribution.[5] This time interval is often referred to as customer "idle" time.

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[3] Yaotung-Lu, op. cit., p. 7

[4] Ibid., p. 7.

[5] Feller, op. cit., pp. 364 - 367.

Yaotung-Lu made several assumptions when he reviewed the arrival segment of the problem. In order to derive the steady state distributions, which will be discussed later in this chapter, he stated that the average arrival rate must be relatively stable over a period of time. If they are stable, the derived average values will be independent of time.[6] To achieve this desired consistency in the arrival distribution, he surveyed the daily sales volume in the store. As the sales fluctuated so did the average arrival rate. Based on this discovery, he proceeded to divide the week into five sub-classes. He then observed the number of arrivals per one minute interval for each of the sample periods.

He recognized the fact that management had little control over the rate at which customers arrived at the store. That is, the store is a part of a much larger system. While this concept has a logical basis, he failed to employ this reasoning consistently throughout his study.

Based on the assumptions made by Yaotung-Lu and a review of the Edmonton supermarket under study, the following hypothesis is offered here:

<u>NULL HYPOTHESIS I</u>. The number of customers arriving at the check-out counters, per unit of time, is generated by a Poisson variable. Thus, the length of time between two successive arrivals is distributed by the negative exponential function.

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[6]Yaotung-Lu, <u>op</u>. <u>cit</u>., p. 13.

3: ANALYSIS OF THE SERVICE DISTRIBUTION--YAOTUNG-LU

Yaotung-Lu defined the service time as the time which elapses while a particular customer is being served. To apply his model he assumed that all check-out counters had identical service mechanisms. He then assumed that the service rate at the check-out counters would be one of two rates. One service rate applied when the cashier operated the check-out counter alone; the other rate applied when the cashier was assisted by a bag boy.

To arrive at the two levels of service time, he again made several assumptions. First, he assumed that the two levels of service time were consistent throughout the week. That is, the average service time per customer was constant throughout the week for each type of serving unit: cashier alone, and cashier and bag boy. When he attempted to optimize his problem, he employed a composite average service time based on the cashier alone and the cashier and bag boy service time distributions.

Second, he used the negative exponential distribution even though his data did not fit this distribution. This assumption facilitated the mathematics required in his study. He did ascertain that the data he obtained from the supermarket in Detroit was approximated by a gamma distribution.[7]

The third assumption was based on the physical capacity of the supermarket under study. He stated that the number of check-out counters made available to the customers at any given time would be

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[7] Ibid., p. 29

less than or equal to the number of check-out counters that the supermarket had at the outset of the study.[8] That is, the check-out facilities could not be altered in the short tun.

He restricted his study by ignoring the express counter in the Detroit store. He felt that the volume of daily sales was small at this counter even though the counter was open to customers all day. This implied that the size of the order was not related to the service time.

A further restriction entered his study when he stated that the presence of a long queue had no effect on the speed of service.

Based on the review of the service time discussion set forth by Yaotung-Lu, the following hypotheses are offered:

<u>NULL HYPOTHESIS II</u>: The service time distribution for servicing customers is not distributed by the negative exponential function, but rather, by some other function (Erlangian type which is a member of the Gamma family).

<u>NULL HYPOTHESIS III</u>. There are significant differences in the average service times within a week.

<u>NULL HYPOTHESIS IV</u>. The differences in the average service time within a week do not occur on a random basis, but rather, follow a predictable pattern within each week.

<u>NULL HYPOTHESIS V</u>. The average service time is a function of the average arrival rate and the average size of order.

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[8] <u>Ibid.</u>, p. 36.

4: THE STEADY STATE DISTRIBUTION--YAOTUNG-LU

Yaotung-Lu assumed that the check-out process will approach a steady state because there are restoring forces within the system which attempt to keep down the length of queue. In many cases, the steady state will give sufficient insight for one to calculate important quantities of the system.[9] This analysis is valid only if the steady state is approximated on the most feasible basis.

To employ the steady state technique, Yaotung-Lu had to make several more assumptions. He presumed that the entire check-out operation was a system. He then stated that the system can be in a number of possible states. The steady state solution gives the probability that the system is in each of the possible states. From these probabilities he calculated the average values of the various quantities of interest: number of customers in the system, the average length of queue, and the derived probabilities that there were more than a certain specified number of customers waiting in each check-out lane.

Certainly the steady state solution technique employed by Yaotung-Lu was beneficial to the study. However, the steady state solution may be improved if an alteration is made to the underlying distributions upon which the steady state was designed. Hence, the following hypothesis:

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[9] Ibid., p. 10.

<u>NULL HYPOTHESIS VI</u>. The steady state technique can be improved if the underlying service time distribution is calculated on a sub-class basis like the arrival rates.

5: SYSTEM SOLUTION--YAOTUNG-LU

Yaotung-Lu predicted the average length of queue under various combinations of servers and combined this result with the cost of lost customers and the costs of the check-out operators, to optimize the system (as he recognized the system). This was an important combination of the underlying distributions. Management cannot alter the flow of customers into the store, but it can regulate the average service rate of its check-out operation by varying the number of check-out counters to be operated as well as the number of bag boys assisting the cashiers. These were the two variables that Yaotung-Lu felt management would like to adjust in such a way that a given criterion of optimality would be met.

The criterion of optimality adopted by Yaotung-Lu was the minimization of expected costs incurred by the supermarket management in providing check-out services per unit of time. The most relevant portion of these costs can be classified into two types: (1) wages of cashiers and bag boys who are providing the check-out service, and (2) costs incurred by a loss of a customer due to frequent formations of an unnecessarily long queue which is caused by inadequate check-out facilities.[10]

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[10] <u>Ibid</u>., p. 38.

The first kind of cost can be readily estimated. It is independent of the traffic intensity. The second kind of cost is not so easily defined. The problem is to quantify the adverse effect of keeping a customer waiting too long and too frequently for check-out service. The customer may become impatient and decide that he will not return to the store in the future. This cost was assumed by Yaotung-Lu to be a function of the expected length of queue and probably it is reasonable to say that the cost increases at an increasing rate as the expected length of queue becomes larger.[11]

The improvement made on Yaotung-Lu's model lies in the analysis of the service-time distributions. The analysis employed by Yaotung-Lu in the average arrival distribution is retained in this study. The basic change is in the underlying service-time distribution upon which the optimizing solution is based.

6: THE IMPORTANCE OF A SUBSYSTEM

In his analysis, Yaotung-Lu recognized that the arrival rates were influenced by external factors, but he ignored this concept when he considered the average service time per customer. It was established in this study that the average service time was, in fact, influenced by factors other than the manpower or the physical service facility. The

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[11] Ibid., p. 38.

size of order and the average arrival rate were influencing factors on the average service time. It is important to note that the grocery store is part of a much larger system. Yaotung-Lu recognized this fact when he stated that the average arrival rate was out of the control of management. He then assumed that the average service time per customer was one of two rates (depending whether the order was checked by a cashier alone or by a cashier and bag boy together).

When it was established that the management had little control over the arrival rate and the size of order, and that the service rate was found to highly correlated to these two variables, it was then determined that the importance of a subsystem had been underestimated by Yaotung-Lu.

Management can influence consumer purchases by advertising, altering store hours and taking advantage of consumer buying habits. There are implications for a sub-optimizing solution because the check-out process is only a subsystem of a much larger system (store, chain store management, and the community).

Based on the preceeding discussion, the following hypothesis is offered:

NULL HYPOTHESIS VII. The check-out operation, as part of the store, is a subsystem of the store and both the average arrival rate and the average service time per customer are influenced by this condition.

7: SUMMARY

Each of the hypotheses set forth in the preceeding portions of this chapter are tested in Chapter IV. The object was to improve the model advanced by Yaotung-Lu.

The underlying distributions upon which the steady state solution was derived were proven to be influenced by external factors: factors uncontrollable by management. The entire system in which the check-out operation was located was not optimized. Rather, it is important to recognize that the problem was sub-optimized and, therefore, only a sub-optimal solution was found.

CHAPTER III

OPERATING CHARACTERISTICS OF THE EDMONTON SUPERMARKET UNDER STUDY

1: STUDY OF THE DAILY SALES VOLUME

The average customer arrival rate per minute fluctuates within each week. This rate was found to be closely related to the sales volume per unit of time. Inspection of the daily sales volume at the Edmonton supermarket, under study for a period of two months, indicated that the variation was periodic with the periodicity of one week (Appendix A, Table A - 1). This is shown in Figure 2, in which the sales volume for each day of each week in the sample are shown. Each horizontal line indicates the sales volume, by week, for each sample sub-class. A low sales volume at the beginning of the week was contrasted with heavy weekend sales. Sales on Friday appeared to constitute a group by themselves.

Study of this diagram suggested that the week could be subdivided into at least four periods so that within each period the customer arrival rate would be reasonably stable. Yaotung-Lu divided the week into five sub-classes.[1]

In this study the same number of sub-classes are employed. The first three days are analyzed separately. Initially, judging

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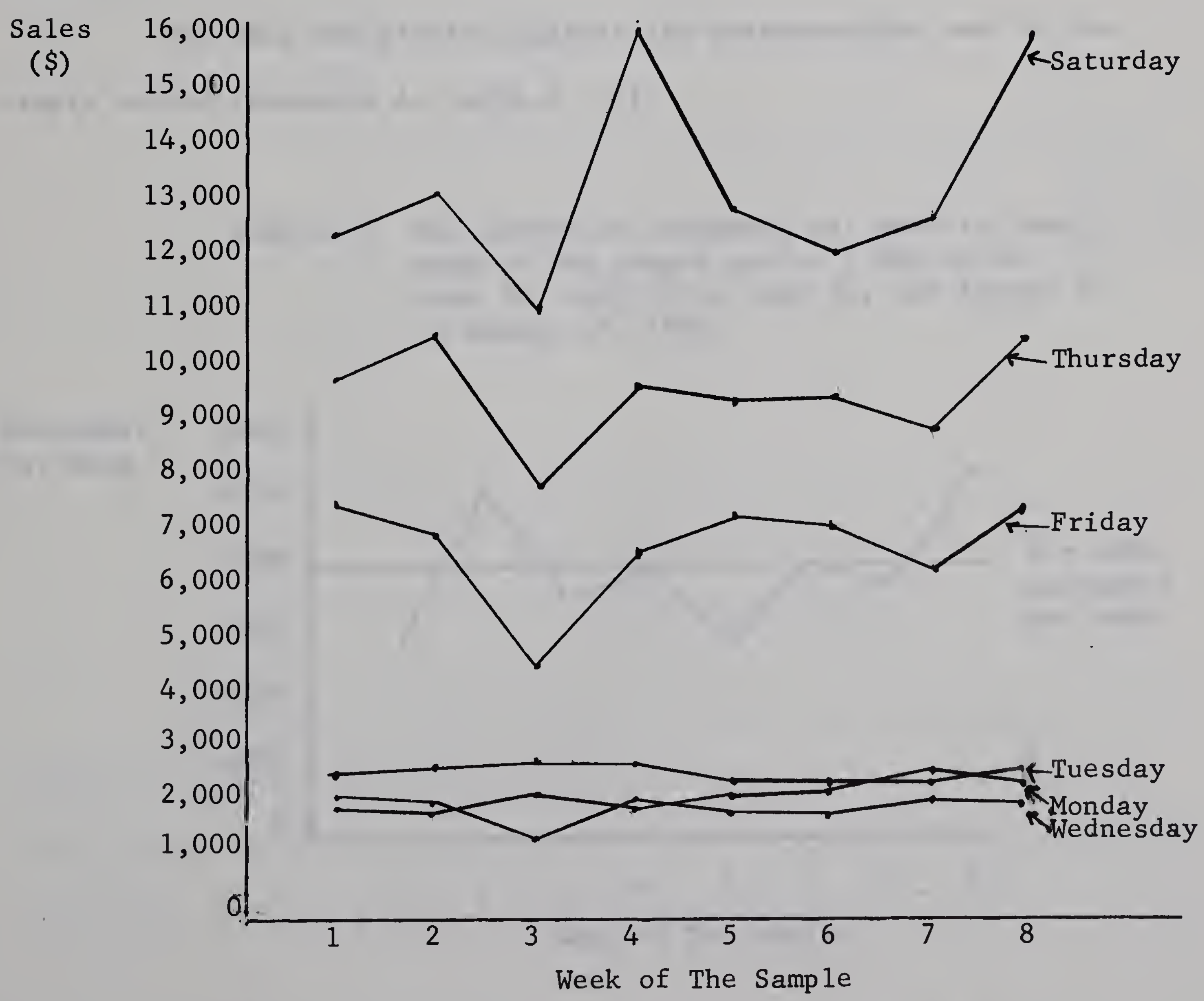
[1]Yaotung-Lu, op. cit., pp. 13 and 15.

from the volumes of the respective sales of Thursday and Saturday, it was conjectured that they would have almost identical traffic intensities. However, it was found that the early part of Thursday was much like the three days at the beginning of the week while the latter part of Thursday appeared to behave like a Saturday. Thursday was then divided into two sub-classes and hence the week was divided into the following five sub-classes: (1) Monday, Tuesday, and Wednesday, (2) Thursday before 5 p.m., (3) Thursday after 5 p.m., (4) Friday, and (5) Saturday.

Figure 2. Daily sales volume at the Edmonton supermarket under study: May 31 to June 26, July 12 to July 31, and August 9 to August 14, 1965.

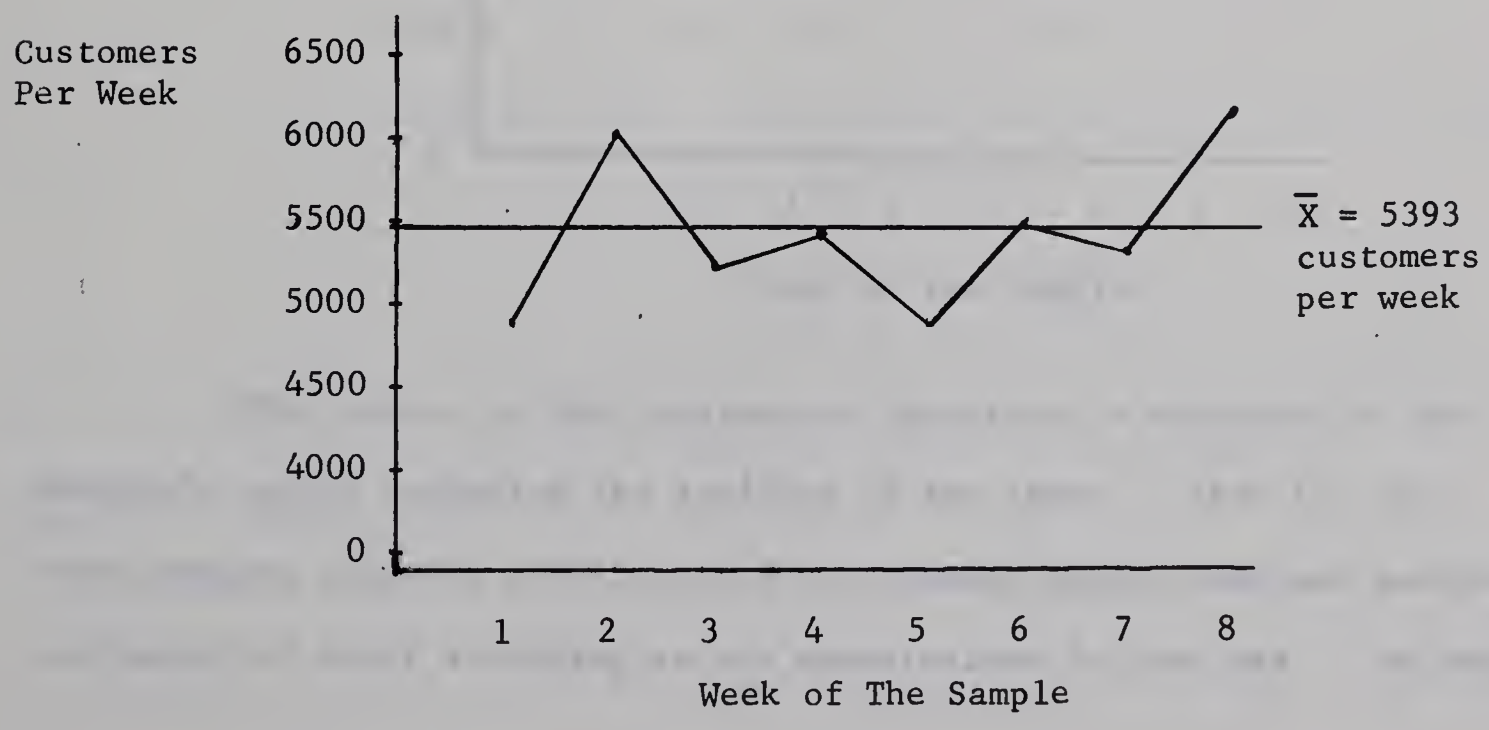


2: THE NUMBER OF CUSTOMERS PER WEEK

Over the duration of the sample period, the number of customers serviced by the store per week did not vary appreciably. Variations in customer traffic intensity are influenced by holidays, store promotion, and by a large number of persons in the labor force receiving their pay cheques at certain intervals. In addition, the third and fourth weeks of every month are influenced by the family allowance cheques. The pay day factor was strongest for the week which included the last day of the month.

The data was plotted against its corresponding week in the sample period (Appendix A, Table A - 2).

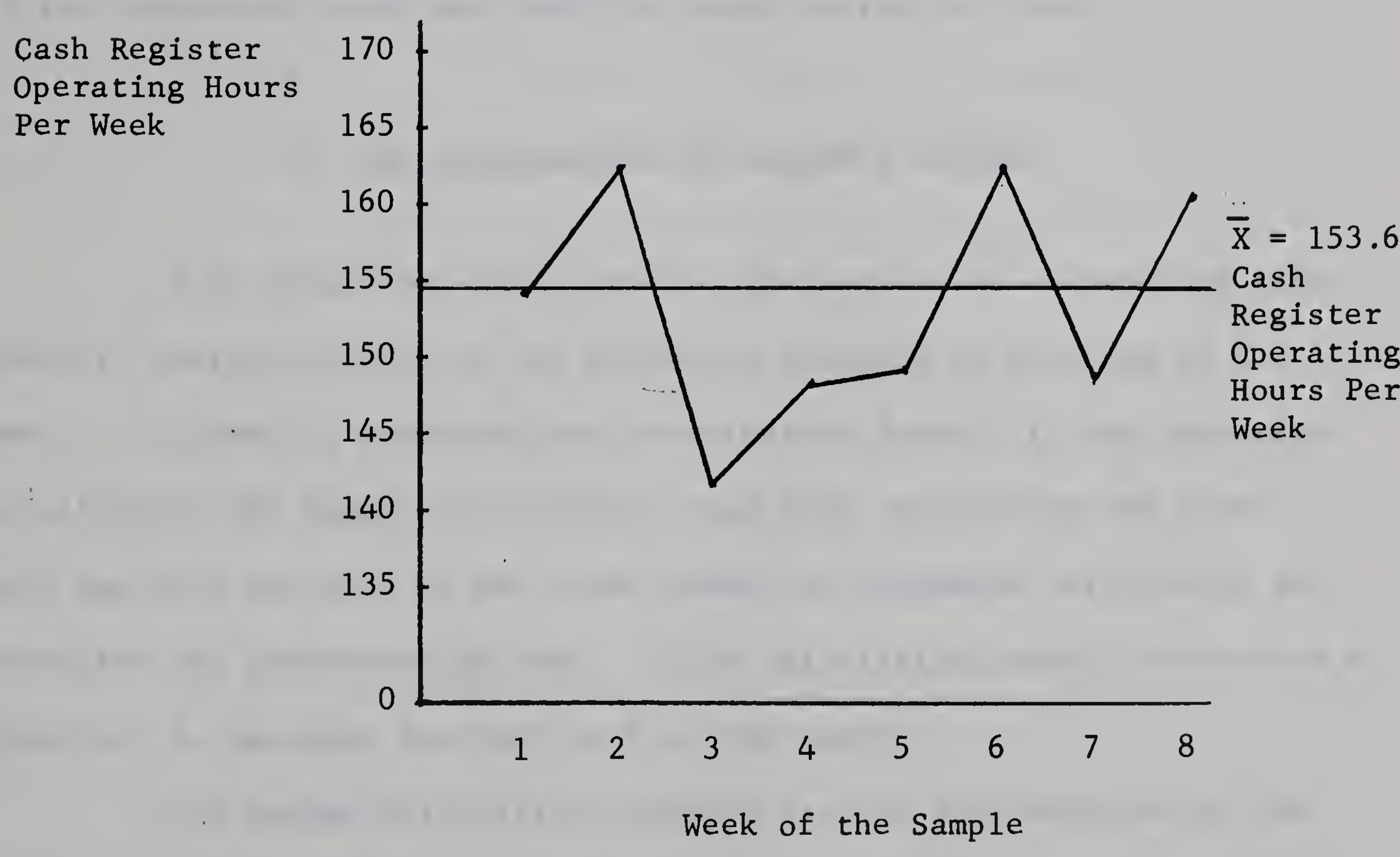
Figure 3. The number of customers per week for each week of the sample period - May 31 to June 26, July 12 to July 31, and August 9 to August 14, 1965.



3: THE NUMBER OF CASH REGISTER OPERATING HOURS PER WEEK

The number of cash register operating hours did not show any marked difference from week to week. The total cash register operating hours per week ranged from 142.8 to 163.2 hours (Appendix A, Talbe A - 3).

Figure 4: The number of cash register operating hours per week for each week of the sample period - May 31 to June 26, July 12 to July 31, and August 9 to August 14, 1965.



This aspect of the supermarket operation is dependent on the manager's policy regarding the staffing of the store. That is, the store manager predicts a daily staff requirement before hand and governs the number of staff according to his expectations for the day. He may

underestimate certain days. When this occurs, he may decide to permit the queues to lengthen during the busy periods rather than attempt to secure addition staff when, in effect, it is too late to do so. This may prove to be detrimental as far as the overall benefits to the store are concerned. That is, if the customer dissatisfaction, arising from long queues, costs the company more money than an additional staff member would, the company would be better off if they had additional staff in the store to service the overflow. The company may be better off even if the additional staff was idle for short periods of time.

4: THE CONTRIBUTION TO BUSINESS FACTOR

This factor was developed for the purpose of determining some specific characteristics of the customers serviced on each day of the week. In order to determine the contribution factor, it was necessary to calculate the number of customers that were serviced by the store each day as a per cent of the total number of customers serviced by the store for the corresponding week. This calculation, shown in Appendix A, Table A - 4, was made for each week of the sample.

The second calculation required was the determination of the daily sales volume as a per cent of the total sales for the corresponding week in the sample. The results of this calculation are shown in Appendix A, Table A - 5.

The contribution to business factor was used to reinforce the decision made earlier in this chapter regarding the number of subclasses into which the week should be divided. The results of this calculation can be found in Appendix A, Table A - 6.

Table A - 6 indicates that the contribution to business factor was very similar for Monday, Tuesday and Wednesday. The factors for Thursday, Friday and Saturday were also of a similar magnitude. The differences between the factors for the first three days and the second three days were an indication of the importance of each day in relation to the overall week. Low business factors at the beginning of the week tended to lower weekly averages while the business factor at the end of the week indicated that Thursday, Friday and Saturday tended to raise weekly averages.

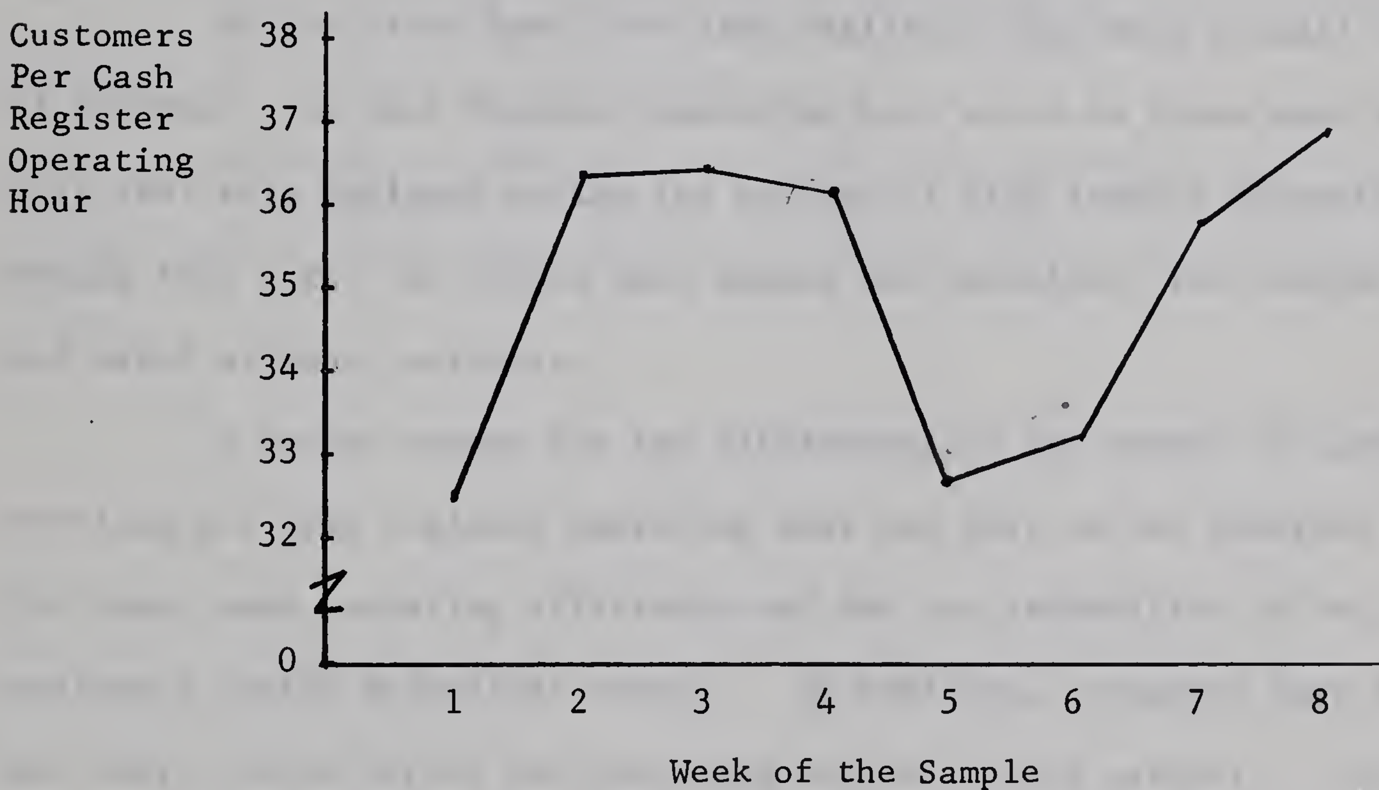
From this review, it was concluded that the sub-classes arrived at earlier in this chapter would be retained for the duration of the study.

5: CUSTOMERS PER CASH REGISTER OPERATING HOUR

The calculation of the number of customers served per cash register operating hour was made by measuring the number of customers that passed through the check-out counters each day. The total number of customers for each day was divided by the number of cash register operating hours for the corresponding day in the sample. In this

calculation, the cash register which serviced the customer was not specified: only the daily totals were used. The results of this calculation may be found in Appendix A, Table A - 7.

Figure 5. The number of customers per cash register operating hour for each week of the sample period - May 31 to June 26, July 12 to July 31, and August 9 to August 14, 1965.



The range of the number of customers per cash register operating hour for weekly totals, was from 32.52 to 37.16 customers per hour (Figure 5). However, there was a wider range (26.48 to 56.67 customers per cash register operating hour) when the data was reviewed on a daily basis (Appendix A, Table A - 7).

6: THE AVERAGE SIZE OF ORDER

It was established earlier that the customers in the first part of the week contributed to a lower business factor. As the average size of the customer orders was smaller in the early part of the week, more customers may be serviced during a one minute interval (Appendix A, Table A - 8).

On the other hand, the cash registers that show a small number of customers per cash register operating hour would be those cash registers that were employed during the periods of high traffic intensity. During this time, the orders were larger and therefore, the cashier could not serve as many customers.

A second reason for the differences in the number of customers serviced per cash register operating hour was that no two cashiers have the exact same operating efficiency and the low probability of two customers having indentical orders. In addition, customers have different habits which affect the operating record of each cashier. For example, some customers forget to select an item and return to the sales floor to pick it up. While the customer is away from the check-out area, the cashier may finish the order and for a short period of time both the cashier and the check-out unit remain idle. A second example was indicated by the slow down that occurs when a customer does not have enough money to pay for his purchases. The cashier must deduct enough items from the bill in order to satisfy the customer and the amount of

money he has for the purchase. The net result was a negative customer per cash register operating hour because the cash register was working in reverse. The cashier spent as much time as she would if she were tallying up an order. The store lost a sale every time she made a deduction.

These examples indicate that there are many imponderables associated with the check-out operation and therefore, direct comparison between two cashiers is very difficult.

In summary, the tables referred to in Appendix A give some insight into the operating characteristics of the supermarket under study.

CHAPTER IV

ANALYSIS

1: ANALYSIS OF THE CUSTOMER ARRIVAL DISTRIBUTIONS

The analysis of the arrival rate distribution was based on the following time distribution: the length of time between two successive customer arrivals.

Empirical probability distributions of the number of customers arriving per unit of time were checked against the theoretical Poisson distributions. The assumption that the customer arrivals per unit of time are distributed by a Poisson distribution implied that the length of time between two consecutive arrivals was distributed by the negative exponential distribution. This assumption was outlined in Chapter II.

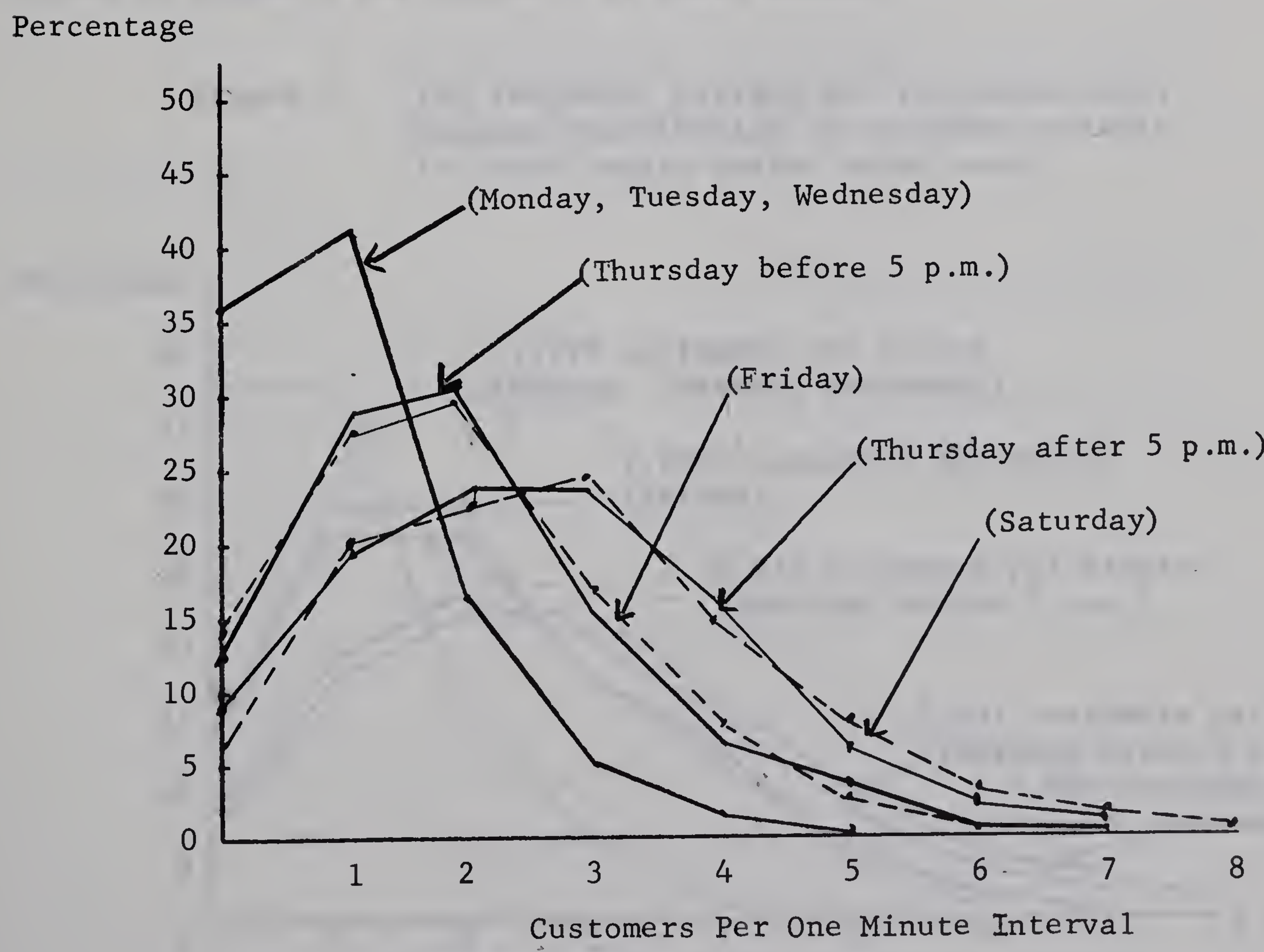
In addition, it was much simpler to count the number of customers arriving at the cash register check-out area per unit of time than to attempt a measurement of the time lapse that occurred between two successive customer arrivals.

Each week in the sample was stratified so that each time period was represented in the final compilations of the 1,470 one minute samples. The stratification of each week into fifteen minute periods was used as a basis for the sampling process. The observations were then carried out by counting the number of customers arriving at the check-out area during a one minute interval.

Goodness-of-Fit Test

Observations, taken for each of the five sub-classes within the eight weeks of the sample, were used to derive the actual frequency distribution. This was done by calculating the occurrence in each arrival class as a percentage of the total frequency observed for the relative sample period (Appendix B, Table B - 7). The percentages were plotted against the arrival classes (Figure 6), and frequently polygons were drawn by connecting the relevant points.

Figure 6. The frequency polygon for the observed distribution of customer arrivals for each sample period under study.



The frequency distributions with the higher traffic volume per unit of time (Thursday after 5 p.m. and Saturday) had a tendency to flatten out in the earlier classes and they had higher frequencies at the right-hand tail. The frequency distributions with lower average traffic volumes per unit of time, peaked earlier and had generally lower frequencies at the right-hand tail.

To compare the observed frequency distributions with the theoretical Poisson distributions, the corresponding theoretical distribution for each group of observations was calculated by using the mean estimated from the observed data with the computer. The resultant distributions are plotted in Figure 7 (Appendix B, Tables B - 2, B - 3, B - 4, B - 5, and B - 6). From sight, the similarity between the curves in Figure 6 and Figure 7 is quite evident.

Figure 7. The frequency polygon for the theoretical Poisson distribution of customer arrivals for each sample period under study.

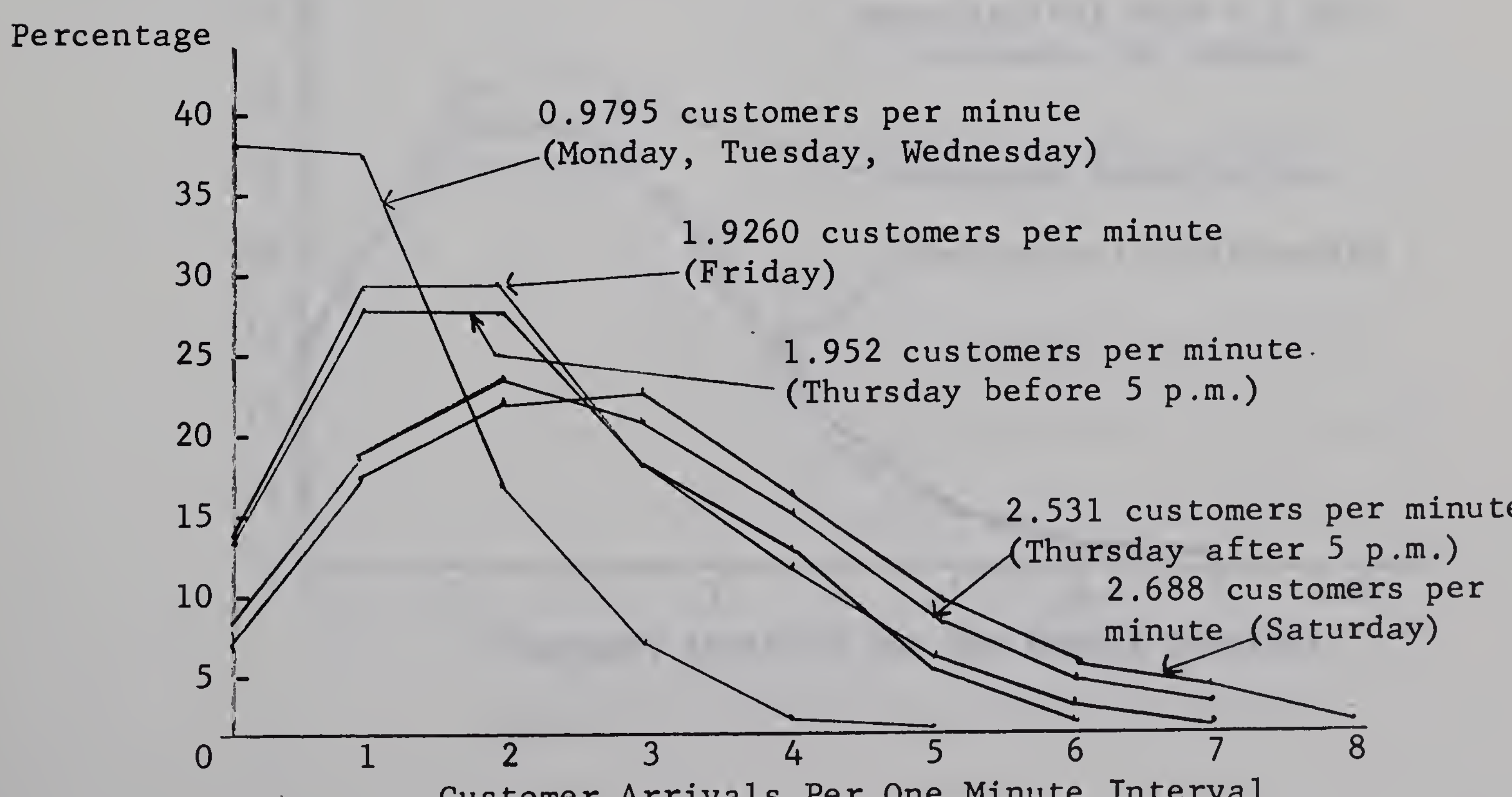


Figure 8. A comparison of the observed and theoretical Poisson frequency distributions for sample period 1: Monday, Tuesday and Wednesday.

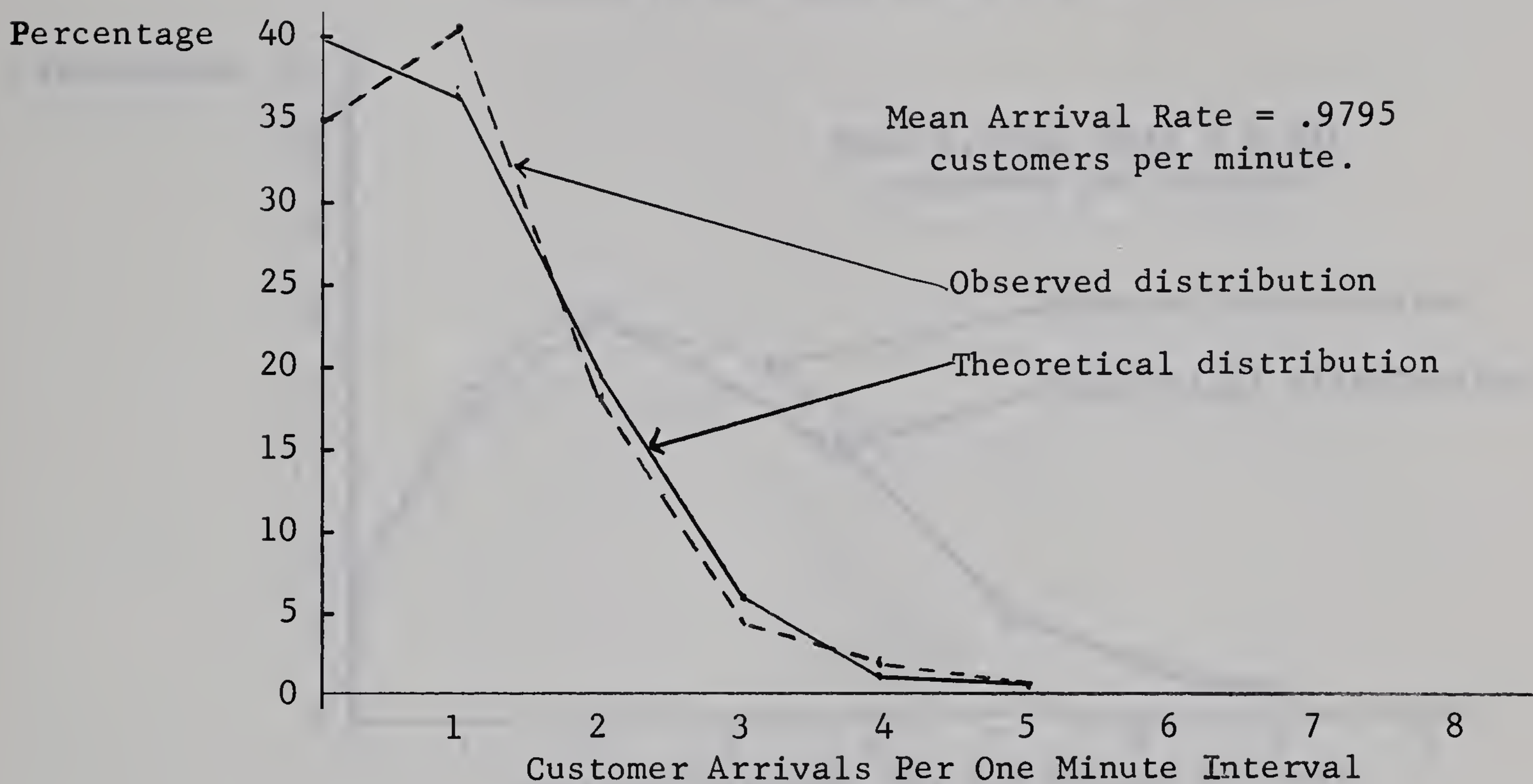


Figure 9. A comparison of the observed and theoretical Poisson frequency distributions for sample period 2: Thursday before 5 p.m.

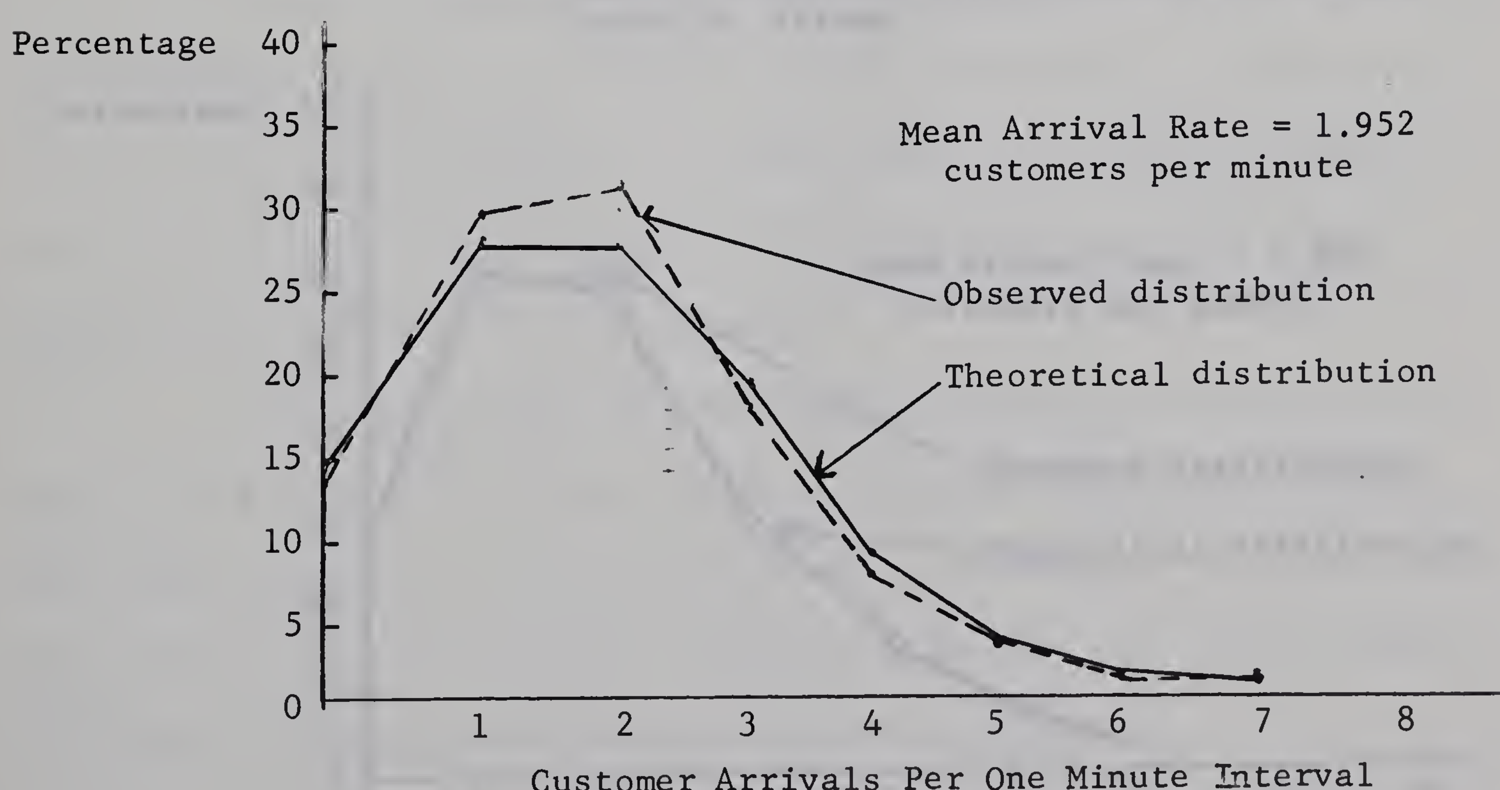


Figure 10. A comparison of the observed and theoretical Poisson frequency distributions for sample Period 3: Thursday after 5 p.m.

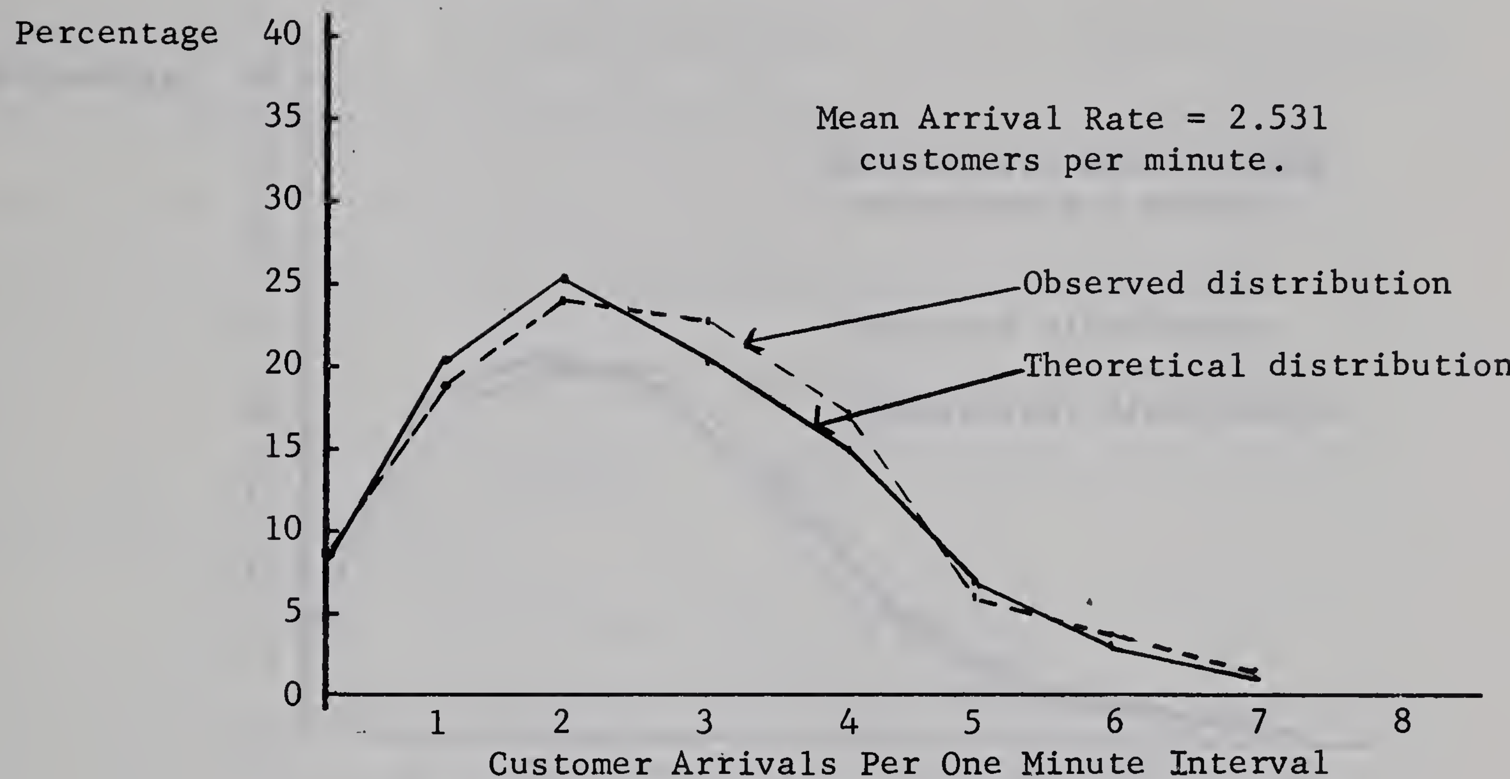


Figure 11. A comparison of the observed and theoretical Poisson frequency distributions for sample period 4: Friday.

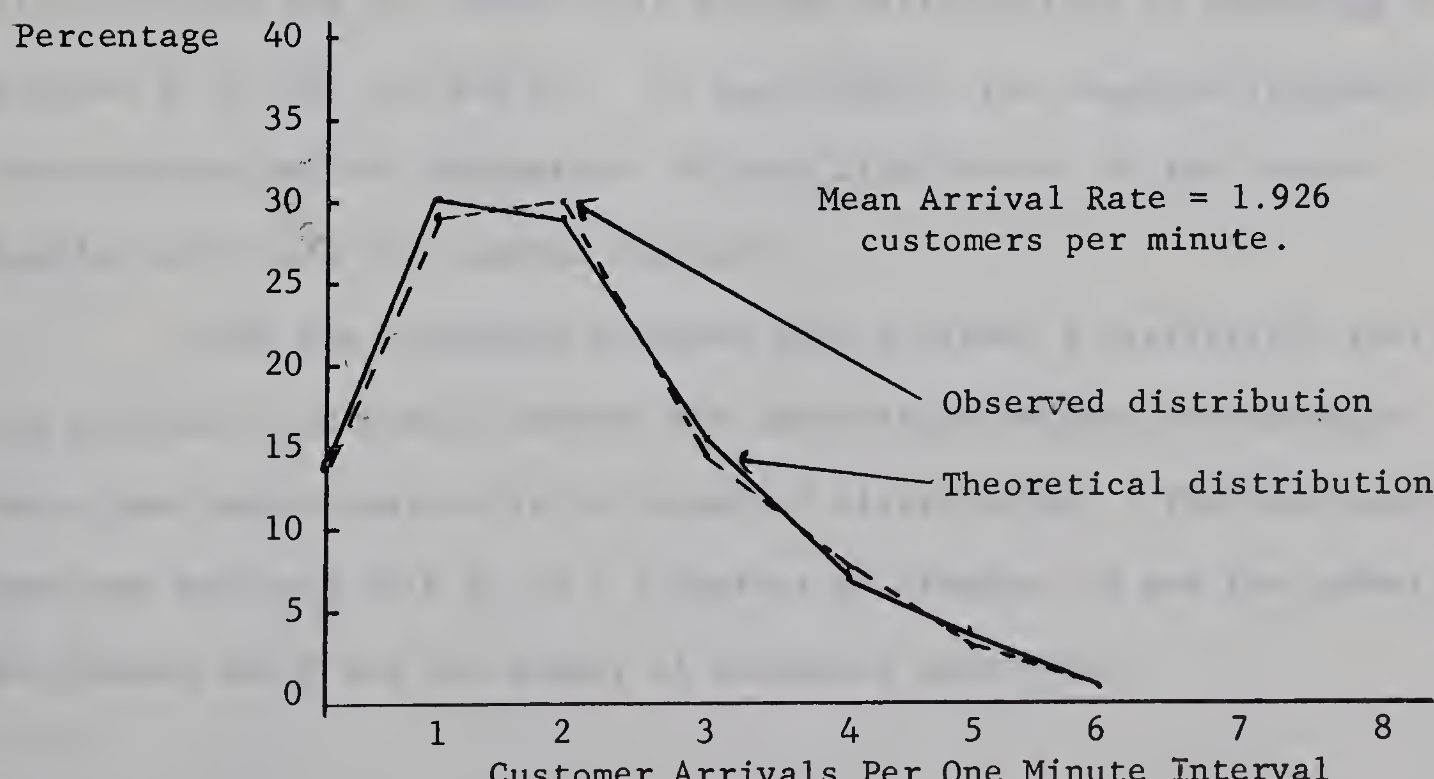
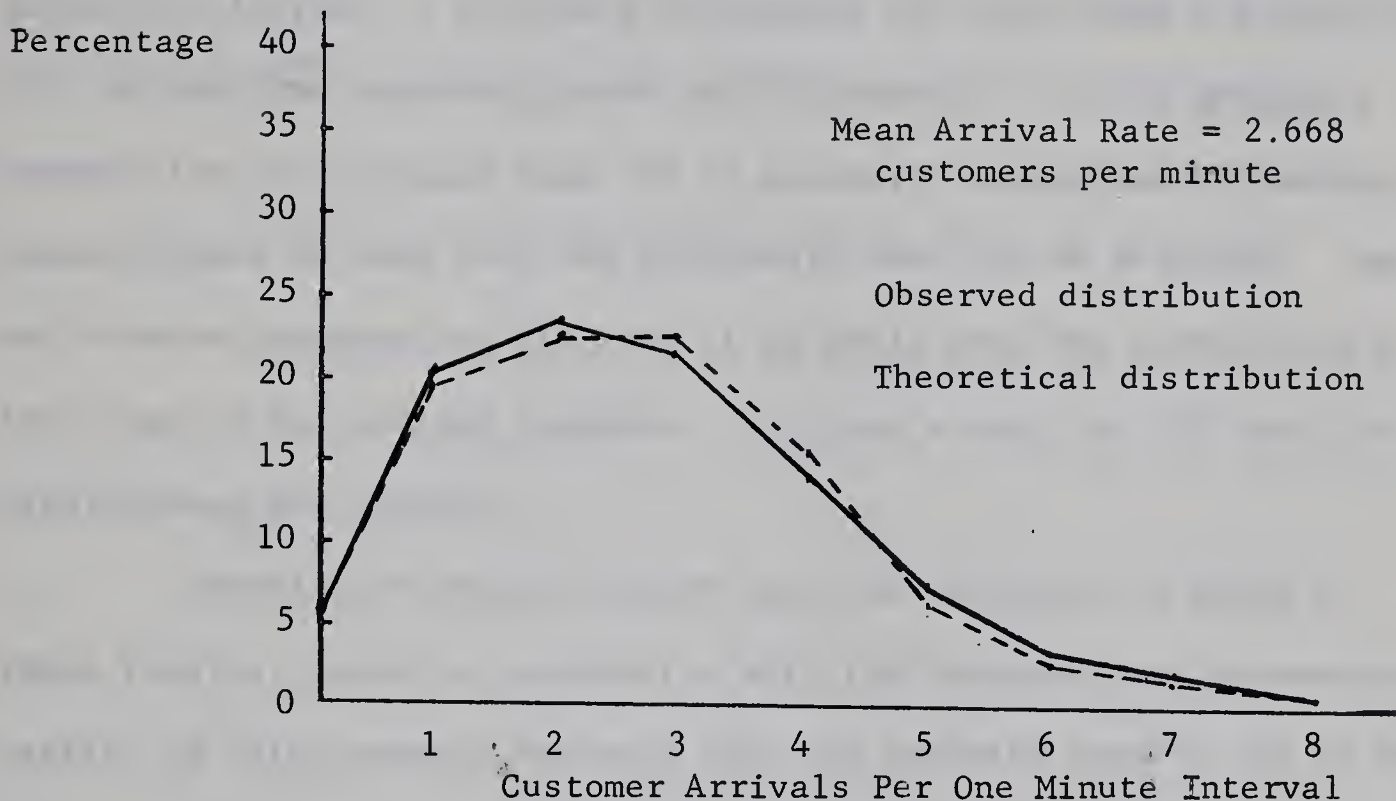


Figure 12. A comparison of the observed and theoretical Poisson frequency distributions for sample period 5: Saturday.



An easier comparison can be made between the observed frequency distributions and the theoretical Poisson distributions by referring to Figures 8, 9, 10, 11, and 12. In each Figure, the observed frequency distribution and the theoretical Poisson distribution of the corresponding sub-class are plotted together.

Once the frequency polygons were plotted, a statistical test was applied to determine whether the theoretical Poisson distribution was a good approximation of the observed distribution. The chi-square test was employed with N - P - 1 degrees of freedom: N was the number of classes and P was the number of estimated parameters.

When the chi-square test is employed, it is necessary to make a decision as to the level of significance that will serve as the decision criterion. A perfect chi-square fit would show a probability of 1.00 that the hypothesis need not be rejected. A fit showing a probability level higher than .05 is generally recognized by leading statisticians to mean that the hypothesis need not be rejected. One may reserve judgement on his test if he feels that the probability is too close to the minimum standard. In this study, the .05 level of significance was adopted.

Results of the chi-square test are tabulated in Table I. These results, taken in conjunction with the diagramatical presentation earlier in this chapter, indicate that the incoming traffic was of the Poisson type. That is, the generating function is a Poisson variate.

TABLE I. CUSTOMER ARRIVAL GOODNESS-OF-FIT TEST RESULTS

Period	Average Arrival Rate Per Minute	Number of One Minute Intervals	Chi-square Statistic Poisson	d.f.	The Significance Level at Which Hypothesis is Rejected
Monday Tuesday Wednesday	.9795	281	2.8621	3	42.35%
Thursday (before 5 p.m.)	1.952	210	3.2711	4	51.39%
Thursday (after 5 p.m.)	2.531	220	3.6961	5	59.70%
Friday	1.926	301	.988	4	91.10%
Saturday	2.668	458	2.3618	6	88.20%

Therefore, HYPOTHESIS I as stated in Chapter II was accepted. The distribution was also checked against the theoretical normal distribution and the results indicated that the normal distribution is not a good approximation of the observed customer arrival distribution.

2: ANALYSIS OF THE CUSTOMER SERVICE TIME DISTRIBUTIONS

In order to apply his model, Yaotung-Lu simplified the check-out operation in a supermarket. He made the following assumptions in order to simplify the study. First, he considered a supermarket which had a finite number of check-out counters.[1] Second, he assumed that each of the check-out counters had identical service mechanisms.[2] Thus, each check-out counter operated independently of the others. Third, he assumed that each check-out counter can be operated at two levels of average check-out rate, say μ_1 and μ_2.[3] It then followed that the number of check-out counters made available to the customers at any given time, would be equal to or less than the number of check-out counters located in the supermarket at the beginning of the study, and that each counter operated at one of the two service rates.

Based on these assumptions he formulated the following rule:[4] $\frac{m - k}{m}$ x 100 per cent[5] of the customer arrivals would be served by the

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[1]Yaotung-Lu, op. cit., p. 7.

[2]Ibid., p. 7.

[3]Ibid., p. 8.

[4]Ibid., p. 8.

[5]The total number of check-out counters in the store = m: therefore, there are (m - k) check-out counters with the average service rate μ_1 and (k) counters with the average service rate μ_2.

check-out counters with the average service rate μ_1 , and the remaining portion of the customers would be served through the counters with service rate μ_2.[6]

Adoption of this rule reduced the problem to a more familiar case of (m) equivalent channels: that is, each of the (m) check-out counters had the average service rate μ , where,

$$\mu = \frac{(m - k)\mu_1 + k\mu_2}{m}.^{7}$$

As soon as each customer arrives at the check-out area, he moves to any check-out counter free to serve him. If all lanes are occupied, he is assumed to form a hypothetical common queue.[8] Yaotung-Lu held that under the conditions of the supermarket, a queue exists when the number of customers waiting for service in the check-out area exceeds the number of operating check-out counters at any time, and their difference is the length of queue.

At this point it was necessary to challenge the study of the service distributions reviewed by Yaotung-Lu. The major problem appeared to be the failure to review the service mechanism in a manner similar to the method employed in the arrival distribution analysis. That is, although he recognized the need for dividing the week into several sub-classes in order to stabilize the arrival patterns of the supermarket, he did not employ a similar technique in analyzing the

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[6]Yaotung-Lu, _op_. _cit_., p. 8.

[7]_Ibid_., p. 8.

[8]_Ibid_., p. 8.

service distributions. However, Yaotung-Lu assumed that the service mechanism would have one of two service time distributions. Based on this conclusion HYPOTHESES III, IV and V were formulated in Chapter II.

Yaotung-Lu assumed that the service time distribution would be distributed by the negative exponential distribution. Although his data did not fit the negative distribution he assumed that it did for the purposes of conducting the study. HYPOTHESIS II was formulated on this assumption.

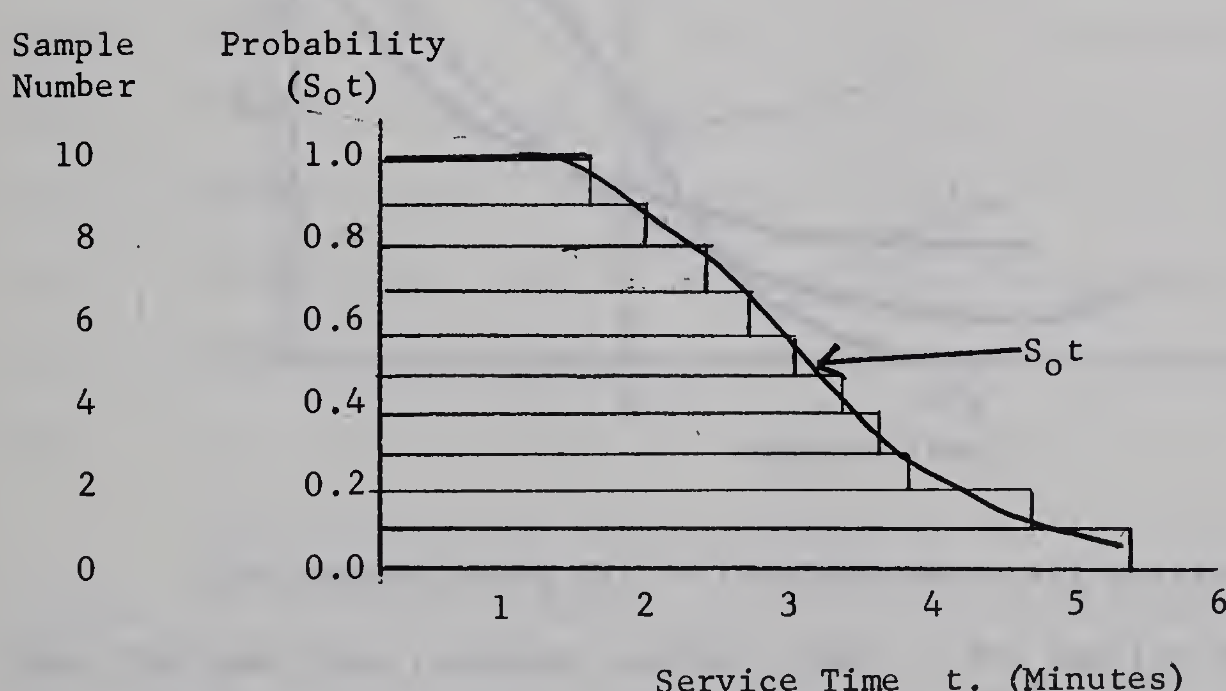
In order to determine the average service time distributions for the supermarket under study, a total of 720 samples were taken. The service time samples were taken from the same sample periods as the arrival distribution data but they did not correspond directly to the arrival distribution samples. That is, each sample period was represented in both time distributions, but those customers sampled for the arrival distribution were not the same customers included in the service distribution data.

The service time data was tested against the negative exponential distribution and was found to be significant at less than one per cent. This was similar to the results obtained by Yaotung-Lu. At this point it was decided that further study of the service time distribution was necessary.

Once it was discovered that the service time distribution did not follow the negative exponential distribution, some difficulty in the determination of the correct theoretical distribution followed.

A graphical technique was employed. A block diagram was constructed by taking each sample of a measured service time and laying the block horizontally on the horizontal axis. The blocks were then arranged vertically in order of length. The vertical axis indicated the sample number and the horizontal axis showed the length of service time. The vertical axis was then converted into a probability range of 0.0 to 1.0. The 1.0 corresponded to the total number of samples under consideration. The following diagram indicates the theoretical presentation discussed above.

Figure 13. The theoretical diagram upon which the graphical presentation employed in this study was based.

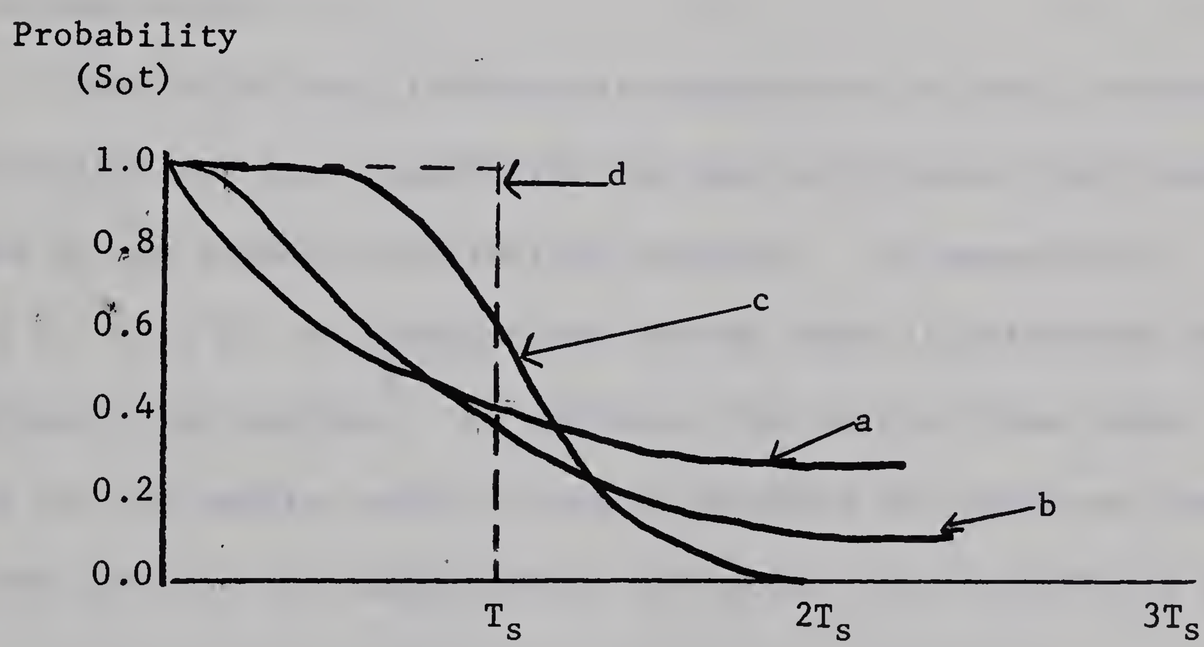


The solid curve is a smoothed-out estimate of the service time distribution ($S_o t$) corresponding to the sample. This is all that is needed for a probabilistic analysis as long as there is no regular

pattern in the occurence of long and short service times. That is, as long as the distribution of the sequence of service durations is random. All curves of $(S_o t)$ start at the point $t = 0$, for it was certain the service would take longer than zero time.

Figure 14 illustrates the various forms that the service time distribution function $(S_o t)$ can take.

Figure 14. The service time distribution function $(S_o t)$; the probability that the service operation takes longer than time t.



The dashed curve (d) is the form where all service operations take the same time (constant service time). The smaller the variation between service times, the nearer the curve will approach curve (d). In the case of curve (d), every unit would take exactly the same service time and therefore, it is certain that the service time would not exceed the time T_s.

At the other end of the spectrum, curve (a) corresponds to a negative exponential service time distribution.

If it is set down that the symbol (k) represents the number of phases in the service operation, then the negative exponential case (curve a) would have a (k) equal to one. Curve (c) would have a (k) greater than one and also a (k) greater than the (k) of curve (b); say a (k) equal to nine or ten.

Curves that take on the shape shown by curves (b) and (c) are often referred to as being Erlangian (Gamma) in nature, and are referred to as Erlang curves.

With this brief theoretical explanation in mind, the service time distributions were graphed for the same sub-classes that were employed in the arrival distribution analysis. In Appendix B, Figures B - 1 to B - 10 indicate the service time distributions for the relevant time periods. In addition, the service times were graphed for the samples where a cashier operated the check-out counter along and also for the samples where the cashier was assisted by a bag boy.

The curves illustrated in Figures B - 1 to B - 10 in Appendix B, appear to be of the Erlangian type. It was not determined in this study whether the value of (k) was five, ten or some other number. Rather, the intention was to explain the curves in terms of the supermarket under study and to show that the curves followed a predictable pattern.

As indicated by the curves in Appendix B, the service time distribution has a (k) greater than one. This indicates that the service mechanism in the supermarket has more than one phase. This appears to be contrary to reason in that the cashier performs the same function for each order. This leads to the conclusion that the phases for a check-out counter would include a (k) of two for a counter operating with a cashier and a bag boy and a (k) of one for a counter operating with a cashier alone. However, when the curves for the cashier alone are reviewed, they appear to have a larger (k) than the curves based on a cashier and a bag boy together. The concept that the servers represented the phases of the service time distribution was not accepted.

The possibility then arose that the phases of service may be related to the number of items in each grocery order. When the graphs are reviewed, this appears to be the case. The orders on Thursday night and Saturday were larger than the orders on Monday, Tuesday and Wednesday. Regardless of whether the counter is staffed by a cashier alone or a cashier and a bag boy, the curves appear to have a larger (k) during periods of increased order size.

Based on the theoretical derivation of the Erlangian curves and the fact that the service time distribution appears to have a (k) greater than one, the HYPOTHESIS that the curves were downward sloping to the right, but Erlangian rather than negative exponential in form, was accepted.

Since the service time distribution curves are Erlangian in nature and appear to have a larger (k) as the size of order increased, it then follows that there must be some significant differences in the average service times between periods within a week.

Thus it becomes a problem of determining whether observed differences between more than two sample means may be reasonably attributed to chance. It was decided that the F-test would be employed to determine whether there was a significant difference between the means for the five sub-classes under study. The analysis was conducted for both the distributions based on a cashier operating alone and a cashier operating with the assistance of a bag boy.

The value of F was calculated by the employment of the following ratio:

$$F = \frac{\dfrac{n \cdot \sum_{j=1}^{k} (\bar{x}_j - \bar{x})^2}{k - 1}}{\dfrac{\sum_{i=1}^{n} \sum_{j=1}^{k} (x_{ij} - \bar{x}_j)^2}{k(n - 1)}}$$

For the distributions of service time related to cashiers operating alone, each of the five sub-classes had sixty-four observations. Therefore, (n) in the formula was equal to sixty-four and (k) was equal to five. In addition, $\bar{x}_j$ was the mean of each sub-class, $\bar{x}$ was the mean of all five sub-classes grouped together, and x_{ij} was the value of each individual observation.

For the service time distribution cashier alone, the value of F was found to be 15.568 with four degrees of freedom in the numerator and three hundred fifteen degrees of freedom in the denominator. The degrees of freedom in the numerator were equal to k - 1 and the degrees of freedom in the denominator were equal to k(n - 1).

In the service time distribution cashier and bag boy, each sample had eighty observations. The same formula was employed with the replacement of (n) equal to sixty-four with an (n) equal to eighty: (k) remained the same.

For the servide time distribution cashier and bag boy, the value of F was found to be 24.258 with four degrees of freedom in the numerator and three hundred ninety-five degrees of freedom in the denominator.

Both values of F, were found to be significant at the .05 and .01 levels of significance. The value of F for four degrees of freedom in the numerator and one hundred twenty degrees of freedom in the denominator was equal to a value 2.45 at the .05 level of significance. The value of F for four degrees of freedom in the numerator and an infinite amount in the denominator was equal to 2.37 at the .05 level of significance. The same argument applied to the .01 level of significance.

The table values of F for the operation cashier alone were 2.37 and 3.32 for the .05 and .01 levels of significance respectively. The calculated value of F was 15.568. The table values of F for the

operation cashier and bag boy were 2.37 and 3.32 for the .05 and .01 levels of significance respectively. The calculated value of F was 24.258. Both tests indicated that the differences between the means of the sub-classes could not be attributed to chance.

For both types of service mechanisms, cashier alone and cashier and bag boy, the mean of the grouped data did not appear to be a good approximation of the sub-class means. Based on this conclusion, HYPOTHESIS III, which stated that there were significant differences between average service times within a week, was accepted.

It was then necessary to determine whether there was a predictable pattern in the behavior of the average service times within a given week.

The average service time per customer, obtained for both service mechanisms for each sub-class for each week of the sample, was compared to the weekly average service time of all samples grouped together. All of the observations for Monday, Tuesday, Wednesday and Thursday before 5 p.m. fell below the grouped average service times for both the cashier alone and the cashier and bag boy service mechanisms. All of the observations for Thursday after 5 p.m. and Saturday fell above the grouped average service times for both service mechanisms. For the sample sub-class Friday, cashier alone, three observations had service times over the weekly average and five had service times below the weekly average. For the Friday sub-class, cashier and bag boy, six observations fell above the weekly average service rate and two fell below the average service rate.

All weekly sub-class averages referred to in this section may be found in the output section of Appendix C. More specifically, the observed value indicated in the "Observed and Estimated Values and Residuals" section of each table show the value that was compared to the weekly average service time for each type of service mechanism.

Based on the finding that the average service times from the sub-classes of the sample fell below the weekly average service time in the first part of the week and above the weekly average service time in the latter part of the week, HYPOTHESIS IV, which stated that the fluctuations in the average service time occurring within a week followed a predictable pattern, was accepted.

It has been established that one; the average service time distribution was Erlangian and not negative exponential, two; that there was a significant difference in the average service time within a week, and three; that this fluctuation of the average service time within the week followed a predictable pattern.

The average service time was influenced by the service mechanisms in that the different service times were in effect when the cashier operated alone as compared to the case where the cashier was assisted by a bag boy. This fact was recognized by Yaotung-Lu.

However, it was concluded that additional factors affected the average service time. If this were not the case, the differences in the average service time within a week would not have been evident. It then followed that some analysis concerning the factors which may affect average service time would have to be made.

TABLE II. THE RESULTS OF THE STUDENT'S t-TEST APPLIED TO THE REGRESSION COEFFICIENTS OBTAINED IN THE COMPUTER RUN.

Sample Period	Service Mechanism	t - value	Degrees of Freedom	Variable Under Consideration	Level of Significance (0.025)[9]
Monday	no bag boy	6.23	6	Size of order	2.447
Thursday before 5 p.m.	no bag boy	15.15	6	Arrival Rate	2.447
Thursday after 5 p.m.	no bag boy	5.32	6	Size of order	2.447
Friday	no bag boy	9.93	6	Size of order	2.447
Saturday	no bag boy	11.35	6	Arrival Rate	2.447
Week Grouped	no bag boy	9.21	38	Size of order	1.960
Monday	bag boy	4.42	6	Size of order	2.447
Thursday before 5 p.m.	bag boy	6.41	6	Arrival Rate	2.447
Thursday after 5 p.m.	bag boy	5.31	6	Arrival Rate	2.447
Friday	bag boy	6.77	6	Arrival Rate	2.447
Saturday	bag boy	6.95	6	Arrival Rate	2.447
Week Grouped	bag boy	10.44	38	Size of order	1.960

[9] The 0.025 level of significance was employed in order to determine the significance of a two-tailed test.

Based on the data obtained at the supermarket in Edmonton, two factors appeared to have some effect on the average service time. These two factors were the average size of order and the average arrival rate.

In order to conduct the analysis, a step-wise regression analysis was performed on the computer. The average service time per customer was set as the dependent variable and the average size of order and the average arrival rate were designated as the independent variables. The correlation was assumed to be linear.

Two tests were used in the analysis. The t-test was used to test whether the regression coefficients were significantly different from zero, and the F-test was used to test the regression mean square for significance.

The results of the t-test are given in Table II. It was found that the average size of order had a strong influence on the average service time per customer. The average arrival rate exerted some influence but not nearly as much as the average size of order.

In all instances, the independent variables were shown to have a significant effect on the dependent variable, in that it was determined that the regression coefficients were significantly different from zero.

It developed that in some cases, the most important variable in the analysis was the average size of order, while in other cases the average arrival rate was more important. The average arrival rate appeared to more important when the cashier was assisted by a bag boy.

It therefore appeared that the size of order was less important when two people were handling the order. The sample size for each of the tests was eight. When the samples were grouped and the total sample size was forth, the average size of order emerged as the most important variable, regardless of whether the customer was serviced by a cashier alone or a cashier assisted by a bag boy.

Based on these results, HYPOTHESIS V, which stated that the average service rate was influenced by the average arrival rate and the average size of order, was accepted.

In all instances, the contribution to variance by the strongest independent variable fell between 69.08 and 97.45 per cent. The contribution to variance by both variables together fell within the range 71.70 to 97.58 per cent. With this in mind it would appear that some other factor, not considered in this study, may be exerting an influence on the average service rate. A second consideration may be that the correlation between the variables was not perfectly linear.

Before leaving this section of the study, some description of the analysis of variance is in order. This analysis was performed with the use of an F-test. The F-ratios for the designated sample groups are shown in Table III.

In all instances, the F-ratio was found to be significant at the .05 level of significance while in two instances, the F-ratio was found to be not significant at the .01 level of significance. Thus, it was indicated that in the total sum of squares of the dependent

TABLE III. THE RESULTS OF THE F-TEST APPLIED TO TEST THE REGRESSION MEAN SQUARE FOR SIGNIFICANCE

Sample Period	Degrees of Freedom in Numerator	Degrees of Freedom in Demoninator	F - Ratio	Value of F for Significant Levels: .05	Value of F for Significant Levels: .01
WITH NO BAG BOY					
Monday	2	5	17.21	5.79	13.3
Thursday (Before 5 p.m.)	2	5	100.59	5.79	13.3
Thursday (After 5 p.m.)	2	5	11.97*	5.79	13.3
Friday	2	5	45.14	5.79	13.3
Saturday	2	5	55.64	5.79	13.3
Week Grouped	2	37	46.88	3.26	4.37
WITH BAG BOY					
Monday	2	5	8.47*	5.79	13.3
Thursday (Before 5 p.m.)	2	5	17.55	5.79	13.3
Thursday (After 5 p.m.)	2	5	13.99	5.79	13.3
Friday	2	5	22.43	5.79	13.3
Saturday	2	5	25.87	5.79	13.3
Week Grouped	2	37	62.61	3.26	4.37

*Not significant at the .01 level of significance.

variable, the reduction due to the combined effect of the two independent variables was not likely the result of chance. The F-ratio, by definition for this section of the study, was employed to test the regression mean square for significance. This further substantiates the case that there are influencing factors exerting some pressure on the service time distribution.

Throughout the analysis of the service time distribution it was intended to ascertain whether the average service time per customer was influenced by factors external to the model as it was designed by Yaotung-Lu.

It was concluded that through the acceptance of HYPOTHESES II to V, there was sufficient evidence that the model set forth by Yaotung-Lu could be modified in order to include the factors discussed in this study.

This point will be discussed further in the next section of this chapter.

3: THE STEADY STATE DISTRIBUTION

Yaotung-Lu was of the opinion the check-out operation would approach a steady state over a period of time. He recognized that the average arrival rate fluctuated during the week, and he therefore divided the week into several sub-classes. The same technique was employed in this study and the results corresponded to those obtained by Yaotung-Lu.

He assumed that the service mechanism would be one of two rates over the week; a rate with a cashier alone and a rate when the cashier was assisted by a bag boy. However, he did not divide the service time distribution analysis into sub-classes. Sufficient evidence has been given in the previous section of this chapter to show that the service time does fluctuate over the week. The service time sample should be divided in a manner similar to the arrival rate analysis.

He built his steady state analysis on the arrival and service time distributions. By dividing the week into five sub-classes for both the average arrival rate and the average service time analyses, the steady state approach would be more realistic. Yaotung-Lu was very careful in dividing the week for the arrival analysis, but he assumed that the week did not need any subdivision for the service time analysis.

It has been shown that the average service time fluctuates widely within a week. If each sample period is considered on its own, it would be involved with fluctuations which occur within the sub-class and not with variations based on the entire week. The steady state would be more of a liklihood if the sub-classification was imposed on the service time distribution.

Based on this brief discussion and the analysis carried out on the service mechanism earlier in this chapter, HYPOTHESIS VI, which stated that the subdivision of the week into sub-classes similar to the arrival analysis would improve the steady state technique, was accepted. This appeared to be a logical conclusion even though it was not tested statistically.

Based on the steady state distribution, Yaotung-Lu determined the average length of queue for various combinations of servers. If a sub-classification of the average service time was implemented one would expect that the predicted average length of queue would be different than if the weekly average was used. Further, Yaotung-Lu attached a cost to the customers lost to the store as a result of growing impatience over persistently long queues. The changes in the classification would have a strong influence on the cost portions of his study. They may indicate that the solution obtained by Yaotung-Lu would be altered under a new system. A comparison of these concepts is illustrated in Chapter V.

As Yaotung-Lu's criterion for optimality was based on the minimization of costs, alterations in the underlying service distributions may have a strong influence on the optimal decision rule. It would also affect the cost of the staff, for the reason that the solution for optimality is based on a composite service time. If the service time methodology is changed, then the number of staff members required at a given time may be altered and the related shifts in employee costs would result.

For the foregoing reasons, it was decided that adoption of a sub-classification in the service time distribution would improve the model. This does not infer that a change in the underlying distributions would upset the method of optimal solution arrived at by Yaotung-Lu.

4: THE IMPORTANCE OF A SUBSYSTEM

When Yaotung-Lu reviewed the arrival rate distribution, he recognized that the customer arrivals were influenced by external factors. He failed to recognize this fact when he reviewed the service time distribution. It has been established that the average service time is influenced by the average arrival rate and the average size of order. It follows that the average arrival rate may be influenced by factors such as advertising, buying habits of the customers, and the store hours during which the customers shop. Therefore, the average service rate may be influenced indirectly by advertising or by the buying habits of the customers. For example, advertising may influence the number of persons who will come into the store, or the price of the items may encourage them to buy more goods. If this is the case, the service time would increase as there would be larger orders and possibly a congestion problem.

Because the average size of order influences the average service time per customer, and the size of order can be influenced by advertising, store hours, holidays and commodity prices, HYPOTHESIS VII, which states that the check-out operation is part of a larger system, was accepted.

5: SUMMARY

In conclusion it may be stated that the average arrival rate is approximated by the Poisson distribution. The average service rate is dependent on several factors such as the average size of order, the average arrival rate, advertising, store hours, buying habits, congestion and management.

The fact that the check-out operation in a supermarket is a subsystem operating within a larger system illustrates the importance of recognizing the optimization for what it is: sub-optimization at best.

Based on the conclusions reached in this chapter, a comparison of the two studies is offered in Chapter V.

CHAPTER V

CHECK-OUT RULES FOR THE SUPERMARKET UNDER STUDY

The purpose of this discussion is to compare the check-out rules for the supermarket under study by basing the analysis on the queuing model developed by Yaotung-Lu and on the modified queuing model introduced in this study.

In order to decide on the grade of check-out service which should be provided to its customers by the store management, one needs more than the knowledge of the probability structure of the queuing process.[1] Thus, Yaotung-Lu introduced the concept of cost into his study. He assumed that once the probability structure had been derived, a cost structure could be superimposed on the queuing process.

As stated earlier, management has little control over the inflow of customers into the store. Secondly, management cannot in the short run, alter the number of check-out counters available to the check-out operation. The seven check-out counters in the store under study are a fixed unit in the short run, and they did not add or eliminate any counters during the course of this study.

Management can, however, regulate the average service rate of the store as a unit by varying the number of check-out counters in

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[1]Yaotung-Lu, op. cit., p.36.

operation as well as by varying the number of bag boys assisting the cashiers. These are in fact two controllable variables that management can adjust in order to minimize the time consumed in the check-out operation in a supermarket.

The hiring of the necessary staff in order to operate all the check-out counters in the store was considered out of the scope of this study just as it was considered out of the scope of Yaotung-Lu's study. If management did not have the required staff to optimize the check-out operation, it would be their responsibility to secure the necessary employees.

The criterion of optimality adopted by Yaotung-Lu was the minimization of expected costs incurred by the supermarket for providing a certain level of check-out service per unit of time.

Yaotung-Lu placed some simplifying assumptions on the study when he applied the above optimal criterion. First, he stated that many supermarkets have express counters to accomodate those customers that have relatively small orders. However, he assumed that the daily sales volume at this counter was very low, and despite the fact that this counter operated most of the day, he ignored it in his study. The supermarket under study in this dissertation did not have an express counter. The results of the tests that related the size of order to the service time indicated that this counter was in fact important to the study.

A second assumption was that there were fewer bag boys than cashiers at any given time. This assumption presented no problem in the present study as it was in fact a policy of the Edmonton supermarket.

Yaotung-Lu further assumed that the length of queue had no effect on the speed of service. However in the local situation, management was of a different opinion. Customer-cashier interaction is higher on slow days. Management therefore concluded that the cashier tends to interact very little when long queues are forming at the check-out area. A second reason for the rejection of this concept was that the management will add bag boys when the queues are long. Appendix B establishes that the number of customers serviced per minute increases when the bag boys are added to the system. This being the case the length of queue will have a significant effect on the average service rate for two reasons: (1) the cashier will interact with the customers in a different manner when the queues are long, and (2) the management will staff the store with the bag boys necessary to overcome the long queues.

Having made these assumptions, Yaotung-Lu proceeded to establish optimum check-out rules for the Detroit supermarket.

1: GENERAL SOLUTION

In the general solution, the first step was to calculate a stationary distribution which described the equilibrium probability of the system in each of all possible states.[2] Yaotung-Lu showed that

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[2] Ibid., p. 38.

the stationary distribution does not change with time and is almost always independent of the condition of the system at time zero.[3] The stationary distribution depends on the probability distribution of customers' demand for service and also the rate at which customers are served.[4]

It was with this concept that the present study encountered problems. Although the probability distribution of the arriving customers could be approximated by the Poisson variate as established by Yaotung-Lu, his model reflected the assumption that the average service time was distributed by the negative exponential distribution. The data collected by Yaotung-Lu did not fit the negative exponential distribution nor did the data collected at the Edmonton supermarket. In fact, the distribution appeared to be Erlangian in nature. Because the underlying service function was not described mathematically, one could not determine the equilibrium probability nor the stationary distribution. At this point Yaotung-Lu assumed the service time distribution was negative exponential so that he could carry out the mathematics.

However, it was concluded that the present study could carry the criteria for optimality to a stage where the modifications made in this study could be tested for their implications without having the service time function mathematically described.

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[3] Ibid., pp. 68 - 77.

[4] Ibid., p. 38.

2: GENERAL METHOD OF SOLUTION

The cost structure was constructed by selecting a number of controllable variables. For each selection, there were certain related costs. Wages were the most important cost, and included wages for cashiers and bag boys. In addition, there is the cost incurred by the supermarket when a customer leaves the store to shop elsewhere. The reason that this customer may leave, in terms of the problem under study, would be the presence of long queues during which the customer grows impatient and seeks another store. With considerable justification the customer feels that the long queues are a sign of inefficiency and/or a restricted service policy on the part of the store.

The first cost, that of the employees, was very easy to determine. The wages per person are independent of the traffic intensity. That is, the staff member is on the payroll, and whether the customers arrive at a slow or fast rate will have no bearing on the cashier's or bag boy's pay cheque. This must be qualified to the short run as management will reduce staff if there is a prevalence of unoccupied (idle) staff.

On the other hand, in order to quantify the cost of the lost customer, it must be considered a function of the expected length of queue. It may be reasonable to assume that the cost will increase at

an increasing rate as the queue length increases.[5] That is, it may have some exponential effects as the queue length increases.

Based on this general solution, and the recognized limitation that the mathematical function describing the service time distribution was unknown, the cost study was carried to the point where the stationary distribution was required. The penalty cost for a lost customer was not derived as it would only be introduced when the length of queue was determined. Therefore, at best, a restricted analysis was performed at this stage.

First, it was necessary to define a check-out rule as follows: for a given average arrival rate, any choice of a combination of the controllable variables in order to provide the check-out service is a form of check-out rule. It then followed that a selected rule which minimized the costs of operating the service counter would constitute a sub-optimal check-out rule. The rule would be optimal if the penalty cost relative to lost customers was included in the analysis as well.

Those values of μ at which the cost functions were to be evaluated were calculated by employing the following formula:

$$\mu = \frac{(m - k)\mu_1 + k\mu_2}{m}$$

where,

μ_1 = The average service rate of a cashier operating alone.

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[5] *Ibid.*, p. 38.

μ_2 = The average service rate of a cashier assisted by a bag boy.

The values of μ are found by taking the reciprocal of the length of time it takes to serve a customer. That is, the number of minutes it takes to serve a customer is converted to determine the number of customers that can be served during a one minute interval.

If the model developed by Yaotung-Lu was employed one would use the values of μ_1 and μ_2 for the entire week regardless of the day. However, due to the conclusions reached about the behavior of the service time in this study, the comparative analysis would include a different value for μ_1 and μ_2 for each sample sub-class. That is, in the light of the additional information available regarding the service time distributions for the sub-classes within a week, the weekly averages would not be used as a criteria for optimality. They were calculated only to provide a base to which the revised service time analysis could be compared.

In this study, the estimated values of μ_1 and μ_2 for each sample sub-class are shown in Table IV.

TABLE IV

THE ESTIMATED VALUES OF THE NUMBER OF CUSTOMERS SERVED PER MINUTE, FOR EACH SUB-CLASS OF THE SAMPLE

Sample Period	Service Mechanism	Average Service Time (Minutes per Customer)	Average Service Rate (Customers Per Minute)
			μ_1
Monday, Tuesday, Wednesday	no bag boy	2.3626	.423
Thursday (before 5 p.m.)	no bag boy	2.3744	.421
Thursday (after 5 p.m.)	no bag boy	3.1129	.321
Friday	no bag boy	2.6598	.376
Saturday	no bag boy	3.1558	.317
Week Grouped	no bag boy	2.7331	.366
			μ_2
Monday, Tuesday, Wednesday	bag boy	1.1733	.852
Thursday (before 5 p.m.)	bag boy	1.2139	.824
Thursday (after 5 p.m.)	bag boy	1.6764	.597
Friday	bag boy	1.5514	.645
Saturday	bag boy	1.8295	.547
Week Grouped	bag boy	1.4889	.672

Because the store under study had seven check-out counters, the number of cashiers (m) ranged from 0 to 7.

The next step was to evaluate the parameter ρ which corresponded to each combination of (m) and (k). This was done by dividing the estimated arrival λ for the period under study, by each of the calculated values of μ.[6] The calculated values of ρ could then be used as a preliminary step in eliminating some check-out rules. The criterion adopted for this elimination process was as follows:

$$m > \frac{\lambda}{\mu_{(m,k,)}}.^{7}$$

When (m) was found to be larger than the calculated value of ρ, the decision rule was retained. Thus, when (m) was smaller than ρ, the combination was eliminated from further consideration. That is, those eliminated at this stage would not be considered in a final optimal analysis where the penalty cost would be considered. In this study, all combinations were studied, and those that would be eliminated in a complete optimal analysis were designated.

In the first column of the following series of tables, values of ρ are given. In column two the calculated values of μ are

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[6]The parameter ρ denotes the ratio of the average arrival rate to the average service rate.

[7]Yaotung-Lu, op. cit., p. 44.

shown. In the next two columns the number of cashiers and the number of bag boys are indicated. In column five the operating cost of the check-out service is given. It was calculated by the following formula:

$$(m - k)\, W_1 + kW_2 .^{8}$$

W_1 was the wage of a cashier per minute and W_2 was the combined wage of a cashier and a bag boy per minute. In this study they were 2.60 cents and 4.52 cents respectively. They were based on the average hourly wage of one dollar and fifty six cents for a cashier and an average hourly wage of one dollar and fifteen cents for a bag boy. The store manager felt that the average would be quite representative of the costs over a period of time. It was necessary to average the hourly wages as many staff members were on different salary grids.

As mentioned previously, the length of queue was not determined. Thus, the total variable cost of the system was not derived.

In column six, the relative traffic intensity ρ /m was given. In the final column, those combinations of cashiers and bag boys which are to be eliminated are indicated.

Before presenting the summary tables, the abbreviations that appear in the tables are explained here for clarity.

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[8] *Ibid*., p. 44

$\hat{\rho} = \dfrac{\hat{\lambda}}{\hat{\mu}}$[9] = The ratio of the estimated average arrival rate to the estimated service rate.

m = The number of cashiers.

k = The number of bag boys.

C = The total operating cost: in cents per minute.

$\frac{\rho}{m}$ = The relative traffic intensity.

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[9]The symbol $\wedge$ denotes as estimation.

TABLE V

CHECK-OUT RULES FOR SAMPLE PERIOD 1: MONDAY, TUESDAY, AND WEDNESDAY: BASED ON A WEEKLY AVERAGE SERVICE RATE

$\hat{\lambda} = .9795$

$\hat{\rho}$	$\hat{\mu}$	m	k	C (cents)	$\frac{\hat{\rho}}{m}$	Combinations to be rejected or accepted
2.68	.366	1	0	2.60	2.68	reject
2.68	.366	2	0	5.20	1.34	reject
1.46	.672	1	1	4.52	1.46	reject
2.68	.366	3	0	7.80	.89	accept
1.88	.519	2	1	7.12	.94	accept
2.68	.366	4	0	10.40	.67	accept
2.09	.468	3	1	9.72	.70	accept
1.46	.672	2	2	9.04	.73	accept
2.68	.366	5	0	13.00	.54	accept
2.21	.443	4	1	12.32	.55	accept
1.72	.570	3	2	11.64	.57	accept
2.68	.366	6	0	15.60	.45	accept
2.29	.427	5	1	14.92	.46	accept
1.88	.519	4	2	14.24	.47	accept
1.46	.672	3	3	13.56	.49	accept
2.68	.366	7	0	18.20	.38	accept
2.35	.417	6	1	17.52	.39	accept
2.01	.488	5	2	16.84	.40	accept
1.64	.596	4	3	16.16	.41	accept
2.39	.410	7	1	20.12	.34	accept
2.09	.468	6	2	19.44	.35	accept
1.78	.550	5	3	18.76	.36	accept
1.46	.672	4	4	18.08	.37	accept

TABLE V (Continued)

CHECK-OUT RULES FOR SAMPLE PERIOD 1: MONDAY, TUESDAY, AND WEDNESDAY: BASED ON A WEEKLY AVERAGE SERVICE RATE

$\hat{\lambda}$ = .9795

$\hat{\rho}$	$\hat{\mu}$	m	k	C (cents)	$\frac{\hat{\rho}}{m}$	Combinations to be rejected or accepted
2.16	.453	7	2	22.04	.31	accept
1.88	.519	6	3	21.36	.31	accept
1.60	.611	5	4	20.68	.32	accept
1.97	.497	7	3	23.96	.28	accept
1.72	.570	6	4	23.28	.29	accept
1.46	.672	5	5	22.60	.29	accept
1.81	.541	7	4	25.88	.26	accept
1.58	.621	6	5	25.20	.26	accept
1.67	.585	7	5	27.80	.24	accept
1.46	.672	6	6	27.12	.24	accept
1.56	.628	7	6	29.72	.22	accept
1.46	.672	7	7	31.64	.21	accept

TABLE VI

CHECK-OUT RULES FOR SAMPLE PERIOD 1: MONDAY, TUESDAY AND WEDNESDAY: BASED ON THE AVERAGE SERVICE RATE DERIVED FOR THIS SAMPLE PERIOD.

$\hat{\lambda}$ = .9795

$\hat{\rho}$	$\hat{\mu}$	m	k	C (cents)	$\frac{\hat{\rho}}{m}$	Combinations to be rejected or accepted.
2.32	.423	1	0	2.60	2.32	reject
2.32	.423	2	0	5.20	1.16	reject
1.15	.852	1	1	4.52	1.15	reject
2.32	.423	3	0	7.80	.77	accept
1.54	.638	2	1	7.12	.77	accept
2.32	.423	4	0	10.40	.58	accept
1.73	.566	3	1	9.72	.58	accept
1.15	.852	2	2	9.04	.58	accept
2.32	.423	5	0	13.00	.46	accept
1.85	.530	4	1	12.32	.46	accept
1.38	.709	3	2	11.64	.46	accept
2.32	.423	6	0	15.60	.39	accept
1.92	.509	5	1	14.92	.39	accept
1.54	.638	4	2	14.24	.39	accept
1.15	.852	3	3	13.56	.39	accept
2.32	.423	7	0	18.20	.33	accept
1.98	.495	6	1	17.52	.33	accept
1.65	.595	5	2	16.84	.33	accept
1.32	.745	4	3	16.16	.33	accept
2.02	.484	7	1	20.12	.29	accept
1.73	.566	6	2	19.44	.29	accept
1.44	.680	5	3	18.76	.29	accept
1.15	.852	4	4	18.08	.29	accept

TABLE VI (Continued)

CHECK-OUT RULES FOR SAMPLE PERIOD 1: MONDAY, TUESDAY AND WEDNESDAY: BASED ON THE AVERAGE SERVICE RATE DERIVED FOR THIS SAMPLE PERIOD.

$\hat{\lambda} = .9795$

$\hat{\rho}$	$\hat{\mu}$	m	k	C (cents)	$\frac{\hat{\rho}}{m}$	Combinations to be rejected or accepted.
1.79	.546	7	2	22.04	.26	accept
1.54	.638	6	3	21.36	.26	accept
1.28	.766	5	4	20.68	.26	accept
1.61	.607	7	3	23.96	.23	accept
1.38	.709	6	4	23.28	.23	accept
1.15	.852	5	5	22.60	.23	accept
1.47	.668	7	4	25.88	.21	accept
1.25	.781	6	5	25.20	.21	accept
1.34	.729	7	5	27.80	.19	accept
1.15	.852	6	6	27.12	.19	accept
1.24	.791	7	6	29.72	.18	accept
1.15	.852	7	7	31.64	.16	accept

TABLE VII

CHECK-OUT RULES FOR SAMPLE PERIOD 5: SATURDAY: BASED ON A WEEKLY AVERAGE SERVICE RATE.

$\hat{\lambda} = 2.668$

$\hat{\rho}$	$\hat{\mu}$	m	k	C (cents)	$\frac{\hat{\rho}}{m}$	Combinations to be rejected or accepted.
7.29	.366	1	0	2.60	7.29	reject
7.29	.366	2	0	5.20	3.65	reject
3.97	.672	1	1	4.52	3.97	reject
7.29	.366	3	0	7.80	2.43	reject
5.14	.519	2	1	7.12	2.57	reject
7.29	.366	4	0	10.40	1.82	reject
5.70	.468	3	1	9.72	1.90	reject
3.97	.672	2	2	9.04	1.99	reject
7.29	.366	5	0	13.00	1.46	reject
6.02	.443	4	1	12.32	1.51	reject
4.68	.570	3	2	11.64	1.56	reject
7.29	.366	6	0	15.60	1.22	reject
6.25	.427	5	1	14.92	1.25	reject
5.14	.519	4	2	14.24	1.29	reject
3.97	.672	3	3	13.56	1.32	reject
7.29	.366	7	0	18.20	1.04	reject
6.40	.417	6	1	17.52	1.07	reject
5.47	.488	5	2	16.84	1.09	reject
4.47	.595	4	3	16.16	1.12	reject
6.51	.410	7	1	20.12	.93	accept
5.70	.468	6	2	19.44	.95	accept
4.85	.550	5	3	18.76	.97	accept
3.97	.672	4	4	18.08	.99	accept

TABLE VII (Continued)

CHECK-OUT RULES FOR SAMPLE PERIOD 5: SATURDAY:
BASED ON A WEEKLY AVERAGE SERVICE RATE.

$\hat{\lambda} = 2.668$

$\hat{\rho}$	$\hat{\mu}$	m	k	C (cents)	$\frac{\hat{\rho}}{m}$	Combinations to be rejected or accepted.
5.89	.453	7	2	22.04	.84	accept
5.14	.519	6	3	21.36	.86	accept
4.36	.611	5	4	20.68	.87	accept
5.37	.497	7	3	23.96	.77	accept
4.68	.570	6	4	23.28	.78	accept
3.97	.672	5	5	22.60	.79	accept
4.93	.541	7	4	25.88	.70	accept
4.29	.621	6	5	25.20	.72	accept
4.56	.585	7	5	27.80	.65	accept
3.97	.672	6	6	27.12	.66	accept
4.25	.628	7	6	29.72	.61	accept
3.97	.672	7	7	31.64	.57	accept

TABLE VIII

CHECK-OUT RULES FOR SAMPLE PERIOD 5: SATURDAY: BASED ON THE AVERAGE SERVICE RATE DERIVED FOR THIS PERIOD.

$\hat{\lambda} = 2.668$

$\hat{\rho}$	$\hat{\mu}$	m	k	C (cents)	$\frac{\hat{\rho}}{m}$	Combinations to be rejected or accepted.
8.41	.317	1	0	2.60	8.41	reject
8.41	.317	2	0	5.20	4.21	reject
4.88	.547	1	1	4.52	4.88	reject
8.41	.317	3	0	7.80	2.80	reject
6.18	.432	2	1	7.12	3.09	reject
8.41	.317	4	0	10.40	2.10	reject
6.77	.394	3	1	9.72	2.26	reject
4.88	.547	2	2	9.04	2.44	reject
8.41	.317	5	0	13.00	1.68	reject
7.11	.375	4	1	12.32	1.78	reject
5.67	.470	3	2	11.64	1.89	reject
8.41	.317	6	0	15.60	1.40	reject
7.35	.363	5	1	14.92	1.47	reject
6.18	.432	4	2	14.24	1.55	reject
4.88	.547	3	3	13.56	1.63	reject
8.41	.317	7	0	18.20	1.20	reject
7.52	.355	6	1	17.52	1.25	reject
6.52	.409	5	2	16.84	1.30	reject
5.45	.490	4	3	16.16	1.36	reject
7.62	.350	7	1	20.12	1.09	reject
6.77	.394	6	2	19.44	1.13	reject
5.86	.455	5	3	18.76	1.17	reject
4.88	.547	4	4	18.08	1.22	reject

TABLE VIII (Continued)

CHECK-OUT RULES FOR SAMPLE PERIOD 5: SATURDAY:
BASED ON THE AVERAGE SERVICE RATE
DERIVED FOR THIS PERIOD.

$\hat{\lambda} = 2.668$

$\hat{\rho}$	$\hat{\mu}$	m	k	C (cents)	$\frac{\hat{\rho}}{m}$	Combinations to be rejected or accepted.
6.96	.383	7	2	22.04	.99	accept
6.18	.432	6	3	21.36	1.03	reject
5.33	.501	5	4	20.68	1.07	reject
6.41	.416	7	3	23.96	.92	accept
5.67	.470	6	4	23.28	.95	accept
4.88	.547	5	5	22.60	.98	accept
5.96	.448	7	4	25.88	.85	accept
5.24	.509	6	5	25.20	.87	accept
5.54	.481	7	5	27.80	.79	accept
4.88	.547	6	6	27.12	.81	accept
5.19	.514	7	6	29.72	.74	accept
4.88	.547	7	7	31.64	.70	accept

From the foregoing tables, the sub-optimum rules for sample periods one and five are indicated. While the tables do not continue to optimization, they still provide useful information because they narrowed the range of the selection for the optimal rules. Hence, the rules in the tables may be considered a screening test from which the possible combinations of cashiers and bag boys can be eliminated if they do not pass the condition for convergence $\rho < m$.

In addition, the results indicate the difference in the check-out rule when a weekly average is employed instead of the average service rate for the sample period under study. The following table indicates the sub-optimum rules reached in the foregoing analysis.

TABLE IX

SUB-OPTIMAL CHECK-OUT RULES FOR THE SUPERMARKET UNDER STUDY

Sample Period	Number of Cashiers (m)	Number of Bag Boys (k)	Operating Cost of Service (cents)
Monday, Tuesday, Wednesday (Using an average service rate)	2	1	7.12
Monday, Tuesday, Wednesday (Using the average service rate for the sample period)	2	1	7.12
Saturday (Using an average service rate for the week)	4	4	18.08
Saturday (Using the average service rate for the sample period)	7	2	22.04

The change in the model did not effect the decision rule for the Monday sample periods. The primary reason for this was that the average arrival rate was very low and the differences between the various levels of the check-out rules were very small. However, when one reviews the sample period for Saturday it becomes evident that the use of an overall weekly average instead of the average service rate corresponding to that sample period changes the check-out rule.

It is important to note that the differences in the two methods of obtaining check-out rules occurred in the sample period which had a higher traffic intensity and the average size of order was greater. The check-out rules in sample period 5 are related to a higher operating cost per minute. Yaotung-Lu's model had a tendency to under estimate the requirements of the store for periods where there were large customer inflows. The very small differences that might have occurred during the lower traffic periods have a negligible effect on the operating costs of the supermarket. However, it was established that the difference in the operating costs at high traffic intensity periods was of the magnitude of 4 cents per minute. This can amount to thousands of dollars per year when one reviews the same problem for all stores in the organization.

Therefore, it was concluded that the use of an average service time for the appropriate sub-class substantially improves the model. This was evident when the computer results were reviewed, but the foregoing exercise illustrated that adoption of this rule would have a significant impact on the solution of the problem and hence on the optimal or sub-optimal check-out rules advanced by Yaotung-Lu.

CHAPTER VI

SUMMARY OF RESULTS AND SUGGESTIONS FOR FURTHER STUDY

A search of the queuing theory literature revealed that very little had been done in the analysis of the supermarket check-out service. A model, constructed by a doctoral student at Michigan State University, appeared to offer a complete analysis of the supermarket problem.

The purpose of this study was to modify the queuing model developed by John Yaotung-Lu in 1959. His model involved the balancing of the costs incurred by the operation of a certain number of check-out counters for a given time period, against the cost of losing customers in the future.

Yaotung-Lu's work established that the functional relationships existing between the various costs, the number of cashiers, the number of bag boys, and the rate at which customers are served could be defined in a logical manner.

His model was based on the arrival distribution being characterized by a Poisson variate, with the service time following the negative exponential distribution. Yaotung-Lu determined that the arrival rate at the Detroit supermarket check-out counters could be approximated by the Poisson distribution, and the same is true in the Edmonton supermarket. Hence, it was concluded that the Poisson distribution best characterized the arrival pattern at supermarket check-out counters.

Despite the fact that the data collected in the Detroit supermarket did not fit the negative exponential distribution, Yaotung-Lu assumed that it did for the purposes of analyzing his model.

In the present study the data did not fit the negative exponential distribution, and it was concluded that some additional analysis of the service time distribution was necessary. The data was graphed in order to obtain a better visual understanding of the actual service time distribution. The results, contained in Appendix B, show that the service time distribution curves are downward sloping to the right, but Erlangian (Gamma) rather than negative expontial. The mathematical formula for each service time distribution was not derived in this study.

Based on the conclusion that the service time distribution curves are Erlangian, it was surmised that some factor or factors must be exerting some influence on the distribution. A step-wise regression analysis was performed and it was established that the service time required per customer highly correlated in each sub-class to the average arrival rate and the average size of order for the corresponding sub-class. It was decided therefore, that the service time analysis should be divided into sub-classes similar to the classes employed in the arrival analysis.

The present study further established that the average service time per customer is significantly different for the various time periods within a week. The sub-optimal check-out rules

developed for sample period five were different when the analysis was based on the weekly average service rate rather than the average service rate for the relevant sub-class. The check-out rule based on the weekly average service rate had a tendency to over-serve in periods of low traffic intensity and under-serve during periods of high traffic flow. When the analysis is based on the service rate derived for the corresponding sub-classes, the variations in service are restricted to fluctuations occurring within the sub-class and not for the entire week.

The steady state analysis conducted by Yaotung-Lu was not tested in this study. A better understanding of the service mechanism is required before this test can be made.

The present study provides a basis for determining a minimum level of check-out service. This solution, while sub-optimal, would appear to offer some valuable guide-lines in making decisions regarding in store service standards. To this extent, the study would appear to have some value to those persons concerned with this problem from a practical point of view.

From the mathematician's point of view, there must be some explanation for the phenomena which influence the service time distributions. A mathematical statement which would represent the service mechanisms is required.

The result of both Yaotung-Lu's study and this study indicate that further analysis of the problem is necessary. Additional study should be concerned with three problems: (1) the relationship between

the variables, (2) the factors which contribute to the missing portion of the variance, and (3) the cost of the lost customer.

The simulated sampling approach is highly recommended for future study of this problem. The advantage of this approach is that more flexibility is available to the analyst and more data can be reviewed in a short period of time. One can simulate the changes in the variables in order to understand the relationships that exist between them.

It is acknowledged that the current study is incomplete, but it does provide an empirical framework for evaluating the check-out operation of a supermarket.

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APPENDICES

APPENDIX A

THE OPERATING CHARACTERISTICS OF

THE SUPERMARKET UNDER STUDY

TABLE A - 1

DAILY SALES VOLUME AT AN EDMONTON SUPERMARKET:
SELECTED WEEKS DURING THE INTERVAL MAY 31 TO AUGUST 14[1]

		Daily Sales Volume in Dollars[2]						
Week	Date[3]	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Total
1	5-31-65	1,930	2,420	1,990	9,680	7,310	12,370	35,700
2	6--7-65	1,840	2,520	1,870	10,500	6,870	13,100	36,700
3	6-14-65	2,200	2,670	1,230	7,770	4,490	11,120	29,480
4	6-21-65	1,860	2,660	1,900	9,550	6,400	16,030	38,400
5	7-12-65	1,960	2,030	1,810	8,960	7,030	13,290	35,080
6	7-19-65	1,810	1,890	1,670	9,010	6,820	12,030	33,230
7	7-26-65	2,210	2,080	1,730	8,380	6,070	13,020	33,490
8	8--9-65	1,830	2,110	1,620	10,210	7,220	16,070	39,060

[1]The weeks June 28 to July 10, and August 1 to August 7, were omitted as they were strongly influenced by holidays (national and local). To this extent, the two weeks were omitted because the holiday reduced the week by one working day.

[2]The daily sales volume was made available to this study on a restricted basis. The agreement stated that the store and location would not be revealed.

[3]The date given is for the Monday of each week in the sample period.

TABLE A - 2

THE NUMBER OF CUSTOMERS PER DAY,
FOR EACH DAY OF EACH WEEK IN THE SAMPLE

Week	Date	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Total
				- customers -				
1	5-31-65	511	789	427	1,036	752	1,461	4,976
2	6--7-65	491	683	529	1,487	1,071	1,654	5,915
3	6-14-65	574	890	388	1,113	892	1,388	5,245
4	6-21-65	501	723	442	1,150	827	1,748	5,391
5	7-12-65	446	765	416	1,033	755	1,485	4,900
6	7-19-65	573	850	391	1,253	926	1,475	5,468
7	7-26-65	651	684	391	1,115	849	1,587	5,277
8	8--9-65	516	722	417	1,533	788	1,996	5,972

TABLE A - 3

THE NUMBER OF CASH REGISTER OPERATING HOURS PER DAY, FOR EACH DAY OF EACH WEEK IN THE SAMPLE

Week	Date	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Total
				- hours -				
1	5-31-65	12.8	22.1	10.5	30.6	28.4	48.6	153.0
2	6--7-65	12.9	19.0	15.1	38.2	30.0	47.5	162.7
3	6-14-65	13.4	17.7	8.0	33.0	27.9	42.8	142.8
4	6-21-65	12.9	20.2	10.5	34.7	22.1	49.1	149.5
5	7-12-65	11.5	19.0	10.9	33.8	23.8	50.3	149.3
6	7-19-65	16.8	23.1	6.9	37.8	27.9	50.7	163.2
7	7-26-65	16.8	20.5	9.0	29.8	25.1	46.5	147.7
8	8--9-65	14.8	21.5	8.4	36.1	26.7	53.2	160.7

TABLE A - 4

THE NUMBER OF DAILY CUSTOMERS AS A PERCENTAGE OF THE TOTAL CUSTOMERS FOR THE WEEK - FOR EACH WEEK OF THE SAMPLE

Week	Date	Day	Customers	Daily Customers as a Percentage of the Total Customers for the Week
1	5-31-65	Monday	511	$\frac{511}{4976} \times 100 = 10.27\%$
		Tuesday	789	$\frac{789}{4976} \times 100 = 15.86\%$
		Wednesday	427	$\frac{427}{4976} \times 100 = 8.58\%$
		Thursday	1036	$\frac{1036}{4976} \times 100 = 20.82\%$
		Friday	752	$\frac{752}{4976} \times 100 = 15.11\%$
		Saturday	1461	$\frac{1461}{4976} \times 100 = 29.36\%$
			4976	100.00%
2	6--7-65	Monday	491	$\frac{491}{5915} \times 100 = 8.30\%$
		Tuesday	683	$\frac{683}{5915} \times 100 = 11.55\%$
		Wednesday	529	$\frac{529}{5915} \times 100 = 8.94\%$
		Thursday	1487	$\frac{1487}{5915} \times 100 = 25.14\%$
		Friday	1071	$\frac{1071}{5915} \times 100 = 18.11\%$
		Saturday	1654	$\frac{1654}{5915} \times 100 = 27.96\%$
			5915	100.00%

TABLE A - 4 (CONTINUED)

THE NUMBER OF DAILY CUSTOMERS AS A PERCENTAGE OF THE TOTAL CUSTOMERS FOR THE WEEK - FOR EACH WEEK OF THE SAMPLE

Week	Date	Day	Customers	Daily Customers as a Percentage of the Total Customers for the Week
3	6-14-65	Monday	574	$\frac{574}{5245} \times 100 = 10.94\%$
		Tuesday	890	$\frac{890}{5245} \times 100 = 16.97\%$
		Wednesday	388	$\frac{388}{5245} \times 100 = 7.40\%$
		Thursday	1113	$\frac{1113}{5245} \times 100 = 21.22\%$
		Friday	892	$\frac{892}{5245} \times 100 = 17.01\%$
		Saturday	1388	$\frac{1388}{5245} \times 100 = 26.46\%$
			5245	100.00%
4	6-21-65	Monday	501	$\frac{501}{5391} \times 100 = 9.29\%$
		Tuesday	723	$\frac{723}{5391} \times 100 = 13.41\%$
		Wednesday	442	$\frac{442}{5391} \times 100 = 8.20\%$
		Thursday	1150	$\frac{1150}{5391} \times 100 = 21.33\%$
		Friday	827	$\frac{827}{5391} \times 100 = 15.34\%$
		Saturday	1748	$\frac{1748}{5391} \times 100 = 32.42\%$
			5391	99.99%

TABLE A - 4 (CONTINUED)

THE NUMBER OF DAILY CUSTOMERS AS A PERCENTAGE OF THE TOTAL CUSTOMERS FOR THE WEEK - FOR EACH WEEK OF THE SAMPLE

Week	Date	Day	Customers	Daily Customers as a Percentage of the Total Customers for the Week
5	7-12-65	Monday	446	$\frac{446}{4900} \times 100 = 9.10\%$
		Tuesday	765	$\frac{765}{4900} \times 100 = 15.61\%$
		Wednesday	416	$\frac{416}{4900} \times 100 = 8.49\%$
		Thursday	1033	$\frac{1033}{4900} \times 100 = 21.08\%$
		Friday	755	$\frac{755}{4900} \times 100 = 15.41\%$
		Saturday	1485	$\frac{1485}{4900} \times 100 = 30.31\%$
			4900	100.00%
6	7-19-65	Monday	573	$\frac{573}{5468} \times 100 = 10.48\%$
		Tuesday	850	$\frac{850}{5468} \times 100 = 15.54\%$
		Wednesday	391	$\frac{391}{5468} \times 100 = 7.15\%$
		Thursday	1253	$\frac{1253}{5468} \times 100 = 22.91\%$
		Friday	926	$\frac{926}{5468} \times 100 = 16.94\%$
		Saturday	1475	$\frac{1475}{5468} \times 100 = 26.98\%$
			5468	100.00%

TABLE A - 4 (CONTINUED)

THE NUMBER OF DAILY CUSTOMERS AS A PERCENTAGE OF THE TOTAL CUSTOMERS FOR THE WEEK - FOR EACH WEEK OF THE SAMPLE

Week	Date	Day	Customers	Daily Customers as a Percentage of the Total Customers for the Week
7	7-26-65	Monday	651	$\frac{651}{5277} \times 100 = 12.34\%$
		Tuesday	684	$\frac{684}{5277} \times 100 = 12.96\%$
		Wednesday	391	$\frac{391}{5277} \times 100 = 7.41\%$
		Thursday	1115	$\frac{1115}{5277} \times 100 = 21.13\%$
		Friday	849	$\frac{849}{5277} \times 100 = 16.09\%$
		Saturday	1587	$\frac{1587}{5277} \times 100 = 30.07\%$
			5277	100.00%
8	8--9-65	Monday	516	$\frac{516}{5972} \times 100 = 8.64\%$
		Tuesday	722	$\frac{722}{5972} \times 100 = 12.09\%$
		Wednesday	417	$\frac{417}{5972} \times 100 = 6.98\%$
		Thursday	1533	$\frac{1533}{5972} \times 100 = 25.67\%$
		Friday	788	$\frac{788}{5972} \times 100 = 13.19\%$
		Saturday	1996	$\frac{1996}{5972} \times 100 = 33.42\%$
			5972	99.99%

TABLE A - 5

THE DAILY SALES VOLUME AS A PERCENTAGE OF THE TOTAL SALES FOR THE WEEK - FOR EACH WEEK OF THE SAMPLE

Week	Date	Day	Daily Business Volume $	Daily Sales Volume as a Percentage of the Total Weekly Sales Volume
1	5-31-65	Monday	1930	$\frac{1930}{35700} \times 100 = 5.41\%$
		Tuesday	2420	$\frac{2420}{35700} \times 100 = 6.78\%$
		Wednesday	1990	$\frac{1990}{35700} \times 100 = 5.57\%$
		Thursday	9680	$\frac{9680}{35700} \times 100 = 27.11\%$
		Friday	7310	$\frac{7310}{35700} \times 100 = 20.48\%$
		Saturday	12370	$\frac{12370}{35700} \times 100 = 34.65\%$
			35700	100.00%
2	6--7-65	Monday	1840	$\frac{1840}{36700} \times 100 = 5.01\%$
		Tuesday	2520	$\frac{2520}{36700} \times 100 = 6.87\%$
		Wednesday	1870	$\frac{1870}{36700} \times 100 = 5.10\%$
		Thursday	10500	$\frac{10500}{36700} \times 100 = 28.61\%$
		Friday	6870	$\frac{6870}{36700} \times 100 = 18.72\%$
		Saturday	13100	$\frac{13100}{36700} \times 100 = 35.69\%$
			36700	100.00%

TABLE A - 5 (CONTINUED)

THE DAILY SALES VOLUME AS A PERCENTAGE OF THE TOTAL SALES FOR THE WEEK - FOR EACH WEEK OF THE SAMPLE

Week	Date	Day	Daily Business Volume $	Daily Sales Volume as a Percentage of the Total Weekly Sales Volume
3	6-14-65	Monday	2200	$\frac{2200}{29480} \times 100 = 7.46\%$
		Tuesday	2670	$\frac{2670}{29480} \times 100 = 9.06\%$
		Wednesday	1230	$\frac{1230}{29480} \times 100 = 4.17\%$
		Thursday	7770	$\frac{7770}{29480} \times 100 = 26.36\%$
		Friday	4490	$\frac{4490}{29480} \times 100 = 15.23\%$
		Saturday	11120	$\frac{11120}{29480} \times 100 = 37.72\%$
			29480	100.00%
4	6-21-65	Monday	1860	$\frac{1860}{38400} \times 100 = 4.84\%$
		Tuesday	2660	$\frac{2660}{38400} \times 100 = 6.93\%$
		Wednesday	1900	$\frac{1900}{38400} \times 100 = 4.95\%$
		Thursday	9550	$\frac{9550}{38400} \times 100 = 24.87\%$
		Friday	6400	$\frac{6400}{38400} \times 100 = 16.67\%$
		Saturday	16030	$\frac{16030}{38400} \times 100 = 41.74\%$
			38400	100.00%

TABLE A - 5 (CONTINUED)

THE DAILY SALES VOLUME AS A PERCENTAGE OF THE TOTAL SALES FOR THE WEEK - FOR EACH WEEK OF THE SAMPLE

Week	Date	Day	Daily Business Volume $	Daily Sales Volume as a Percentage of the Total Weekly Sales Volume
5	7-12-65	Monday	1960	$\frac{1960}{35080} \times 100 = 5.59\%$
		Tuesday	2030	$\frac{2030}{35080} \times 100 = 5.79\%$
		Wednesday	1810	$\frac{1810}{35080} \times 100 = 5.16\%$
		Thursday	8960	$\frac{8960}{35080} \times 100 = 25.54\%$
		Friday	7030	$\frac{7030}{35080} \times 100 = 20.04\%$
		Saturday	13290	$\frac{13290}{35080} \times 100 = 37.88\%$
			35080	100.00%
6	7-19-65	Monday	1810	$\frac{1810}{33230} \times 100 = 5.45\%$
		Tuesday	1890	$\frac{1890}{33230} \times 100 = 5.69\%$
		Wednesday	1670	$\frac{1670}{33230} \times 100 = 5.03\%$
		Thursday	9010	$\frac{9010}{33230} \times 100 = 27.11\%$
		Friday	6820	$\frac{6820}{33230} \times 100 = 20.52\%$
		Saturday	12030	$\frac{12030}{33230} \times 100 = 36.20\%$
			33230	100.00%

TABLE A - 5 (CONTINUED)

THE DAILY SALES VOLUME AS A PERCENTAGE OF THE TOTAL SALES FOR THE WEEK - FOR EACH WEEK OF THE SAMPLE

Week	Date	Day	Daily Business Volume $	Daily Sales Volume as a Percentage of the Total Weekly Sales Volume
7	7-26-65	Monday	2210	$\frac{2210}{33490} \times 100 = 6.60\%$
		Tuesday	2080	$\frac{2080}{33490} \times 100 = 6.21\%$
		Wednesday	1730	$\frac{1730}{33490} \times 100 = 5.17\%$
		Thursday	8380	$\frac{8380}{33490} \times 100 = 25.02\%$
		Friday	6070	$\frac{6070}{33490} \times 100 = 18.12\%$
		Saturday	13020	$\frac{13020}{33490} \times 100 = 38.88\%$
			33490	100.00%
8	8--9-65	Monday	1830	$\frac{1830}{39060} \times 100 = 4.69\%$
		Tuesday	2110	$\frac{2110}{39060} \times 100 = 5.40\%$
		Wednesday	1620	$\frac{1620}{39060} \times 100 = 4.15\%$
		Thursday	10210	$\frac{10210}{39060} \times 100 = 26.14\%$
		Friday	7220	$\frac{7220}{39060} \times 100 = 18.48\%$
		Saturday	16070	$\frac{16070}{39060} \times 100 = 41.14\%$
			39060	100.00%

TABLE A - 6

THE CONTRIBUTION TO BUSINESS FACTOR:
THE RATIO OF THE PERCENTAGE OF THE WEEK'S CUSTOMERS
TO THE PERCENTAGE OF THE WEEK'S BUSINESS - FOR EACH DAY
OF EACH WEEK OF THE SAMPLE

Week	Date	Day	Percentage of the Week's Customers (by day) (1)	Percentage of the Week's Business (by day) (2)	Business Factor (2)÷(1)
1	5-31-65	Monday	10.27%	5.41%	.527
		Tuesday	15.86%	6.78%	.427
		Wednesday	8.58%	5.57%	.649
		Thursday	20.82%	27.11%	1.302
		Friday	15.11%	20.48%	1.355
		Saturday	29.36%	34.65%	1.180
2	6--7-65	Monday	8.30%	5.01%	.604
		Tuesday	11.55%	6.87%	.595
		Wednesday	8.94%	5.10%	.570
		Thursday	25.14%	28.61%	1.138
		Friday	18.11%	18.72%	1.034
		Saturday	27.96%	35.69%	1.276
3	6-14-65	Monday	10.94%	7.46%	.682
		Tuesday	16.97%	9.06%	.534
		Wednesday	7.40%	4.17%	.564
		Thursday	21.22%	26.36%	1.242
		Friday	17.01%	15.23%	.895
		Saturday	26.46%	37.72%	1.426
4	6-21-65	Monday	9.29%	4.84%	.521
		Tuesday	13.41%	6.93%	.517
		Wednesday	8.20%	4.95%	.604
		Thursday	21.33%	24.87%	1.166
		Friday	15.34	16.67%	1.087
		Saturday	32.42%	41.74%	1.287
5	7-12-65	Monday	9.10%	5.59%	.614
		Tuesday	15.61%	5.79%	.371
		Wednesday	8.49%	5.16%	.608
		Thursday	21.08%	25.54%	1.212
		Friday	15.41%	20.04%	1.300
		Saturday	30.31%	37.88%	1.250

TABLE A - 6 (CONTINUED)

THE CONTRIBUTION TO BUSINESS FACTOR:
THE RATIO OF THE PERCENTAGE OF THE WEEK'S CUSTOMERS
TO THE PERCENTAGE OF THE WEEK'S BUSINESS - FOR EACH DAY
OF EACH WEEK OF THE SAMPLE

Week	Date	Day	Percentage of the Week's Customers (by day) (1)	Percentage of the Week's Business (by day) (2)	Business Factor (2)÷(1)
6	7-19-65	Monday	10.48%	5.45%	.520
		Tuesday	15.54%	5.69%	.366
		Wednesday	7.15%	5.03%	.703
		Thursday	22.91%	27.11%	1.183
		Friday	16.94%	20.52%	1.211
		Saturday	26.98%	36.20%	1.342
7	7-26-65	Monday	12.34%	6.60%	.535
		Tuesday	12.96%	6.21%	.479
		Wednesday	7.41%	5.17%	.698
		Thursday	21.13%	25.02%	1.184
		Friday	16.09%	18.12%	1.126
		Saturday	30.07%	38.88%	1.293
8	8--9-65	Monday	8.64%	4.69%	.543
		Tuesday	12.09%	5.40%	.447
		Wednesday	6.98%	4.15%	.595
		Thursday	25.67%	26.14%	1.018
		Friday	13.19%	18.48%	1.401
		Saturday	33.42%	41.14%	1.231

TABLE A - 7

THE NUMBER OF CUSTOMERS PER CASH REGISTER OPERATING HOUR FOR EACH DAY OF THE SAMPLE

Week	Date	Day	Customers (1)	Cash Register Operating Hours (2)	Customers Per Cash Register Operating Hour (1)÷(2)
1	5-31-65	Monday	511	12.8	39.92
		Tuesday	789	22.1	35.70
		Wednesday	427	10.5	40.67
		Thursday	1036	30.6	33.86
		Friday	752	28.4	26.48
		Saturday	1461	48.6	30.06
TOTALS			4976	153.0	32.52
2	6--7-65	Monday	491	12.9	38.06
		Tuesday	683	19.0	35.95
		Wednesday	529	15.1	35.03
		Thursday	1487	38.2	38.93
		Friday	1071	30.0	35.70
		Saturday	1654	47.5	34.82
TOTALS			5915	162.7	36.36
3	6-14-65	Monday	574	13.4	42.84
		Tuesday	890	17.7	50.28
		Wednesday	388	8.0	48.50
		Thursday	1113	33.0	33.73
		Friday	892	27.9	31.97
		Saturday	1388	42.8	32.43
TOTALS			5245	142.8	36.73
4	6-21-65	Monday	501	12.9	38.84
		Tuesday	723	20.2	35.79
		Wednesday	442	10.5	42.10
		Thursday	1150	34.7	33.14
		Friday	827	22.1	37.42
		Saturday	1748	49.1	35.60
TOTALS			5391	149.5	36.06

TABLE A - 7 (CONTINUED)

THE NUMBER OF CUSTOMERS PER CASH REGISTER OPERATING HOUR FOR EACH DAY OF THE SAMPLE

Week	Date	Day	Customers (1)	Cash Register Operating Hours (2)	Customers Per Cash Register Operating Hour (1)÷(2)
5	7-12-65	Monday	446	11.5	38.78
		Tuesday	765	19.0	40.26
		Wednesday	416	10.9	38.17
		Thursday	1033	33.8	30.56
		Friday	755	23.8	31.72
		Saturday	1485	50.3	29.52
TOTALS			4900	149.3	32.82
6	7-19-65	Monday	573	16.8	34.11
		Tuesday	850	23.1	36.80
		Wednesday	391	6.9	56.67
		Thursday	1253	37.8	33.15
		Friday	926	27.9	33.19
		Saturday	1475	50.7	29.09
TOTALS			5468	163.2	33.50
7	7-26-65	Monday	651	16.8	38.75
		Tuesday	684	20.5	33.37
		Wednesday	391	9.0	43.44
		Thursday	1115	29.8	37.42
		Friday	849	25.1	33.82
		Saturday	1587	46.5	34.13
TOTALS			5277	147.7	35.73
8	8--9-65	Monday	516	14.8	34.86
		Tuesday	722	21.5	33.58
		Wednesday	417	8.4	49.64
		Thursday	1533	36.1	42.47
		Friday	788	26.7	29.51
		Saturday	1996	53.2	37.52
TOTALS			5972	160.7	37.16

TABLE A - 8

THE AVERAGE SIZE OF ORDER FOR EACH DAY OF EACH WEEK OF THE SAMPLE PERIOD

Week	Average Size of Order					
	Monday Tuesday Wednesday	Thursday (before 5 p.m.)	Thursday (after 5 p.m.)	Friday	Saturday	Average for the Week
	in dollars					
1	3.67	7.13	12.05	9.72	8.47	7.18
2	3.66	5.39	9.10	6.41	7.92	6.21
3	3.29	5.33	9.00	5.03	8.01	5.62
4	3.85	6.34	10.71	7.74	9.17	7.12
5	3.56	6.63	11.18	9.31	8.95	7.16
6	2.96	5.49	9.27	7.37	8.16	6.08
7	3.49	5.74	9.68	7.15	8.20	6.35
8	3.36	5.09	8.58	9.16	8.05	6.54
Mean as Calculated by the Computer	3.4800	5.8925	9.9462	7.7363	8.3662	7.0843
Variance about the Mean	.0761	.5248	1.5054	2.5714	.2134	5.9385
Standard Deviation about the Mean	.2759	.7244	1.2269	1.6035	.4620	2.4369

APPENDIX B

SAMPLE DATA

TABLE B - 1

THE FREQUENCY DISTRIBUTIONS OF CUSTOMER ARRIVALS FOR EACH SAMPLING PERIOD

Customer Arrivals Per One Minute Interval	Frequency of Occurrence				
	Monday Tuesday Wednesday	Thursday (before 5 p.m.)	Thursday (after 5 p.m.)	Friday	Saturday
0	101	26	19	40	30
1	113	61	39	84	83
2	46	64	53	88	108
3	14	33	52	51	109
4	6	14	37	26	74
5	1	8	13	9	33
6	-	2	5	3	14
7	-	2	2	-	5
8	-	-	-	-	2
TOTALS	281	210	220	301	458

TABLE B - 2

THE FREQUENCY DISTRIBUTION OF CUSTOMER ARRIVALS, THE THEORETICAL POISSON DISTRIBUTION, AND THE CHI-SQUARE GOODNESS-OF-FIT TEST, FOR SAMPLE PERIOD 1: (Monday, Tuesday, Wednesday)

From the computer run:

Sample Period 1: The mean arrival rate = .9795 customers per minute.
The variance = .0127 customers per minute.
The standard deviation = .1126 customers per minute.

Customer Arrivals Per One Minute Interval X	Frequency Observed F_o	Theoretical Poisson Frequency Calculated F_c	$\frac{(F_o - F_c)^2}{F_c} = \chi^2$
0	101	105.54	.1953
1	113	103.43	.8854
2	46	50.68	.4321
3	14	16.52	.3844
4	6)) 7	4.05)) 4.84	.9649
5	1)	.79)	
TOTALS	281	281.01	2.8621 = χ^2

The degrees of freedom = d.f. = N - P - 1 = 5 - 1 - 1 = 3

From the χ^2 table, a level of significance of 30% = 3.665, and a level of significance of 50% = 2.366. Using linear interpolation, a value of χ^2 = 2.8621, and 3 degrees of freedom will yield a level of significance of 42.35%.

TABLE B - 3

THE FREQUENCY DISTRIBUTION OF CUSTOMER ARRIVALS, THE THEORETICAL POISSON DISTRIBUTION, AND THE CHI-SQUARE GOODNESS-OF-FIT TEST, FOR SAMPLE PERIOD 2: (Thursday before 5 p.m.)

From the computer run:

Sample Period 2: The mean arrival rate = 1.952 customers per minute.
The variance = .0154 customers per minute.
The standard deviation = .1242 customers per minute.

Customer Arrivals Per One Minute Interval X	Frequency Observed F_o	Theoretical Poisson Frequency Calculated F_c	$\frac{(F_o - F_c)^2}{F_c} = \chi^2$
0	26	29.841	.4944
1	61	58.261	.1288
2	64	56.874	.8929
3	33	37.013	.4351
4	14	18.066	.9151
5	8)	7.054)	
6	2) 12	2.295) 9.989	.4048
7	2)	.640)	
TOTALS	210	210.04	3.2711 = χ^2

The degrees of freedom = d.f. = N - P - 1 = 6 - 1 - 1 = 4

From the χ^2 table, a level of significance of 70% = 2.195, and a level of significance of 50% = 3.357. Using linear interpolation, a value of χ^2 = 3.2711 and 4 degrees of freedom will yield a level of significance of 51.39%.

TABLE B - 4

THE FREQUENCY DISTRIBUTION OF CUSTOMER ARRIVALS,
THE THEORETICAL POISSON DISTRIBUTION, AND THE
CHI-SQUARE GOODNESS-OF-FIT TEST,
THE SAMPLE PERIOD 3: (Thursday after 5 p.m.)

From the computer run:

Sample Period 3: The mean arrival rate = 2.531 customers per minute.
The variance = .0133 customers per minute.
The standard deviation = .1152 customers per minute.

Customer Arrivals Per One Minute Interval X	Frequency Observed F_o	Theoretical Poisson Frequency Calculated F_c	$\frac{(F_o - F_c)^2}{F_c} = \chi^2$
0	19	17.534	.1225
1	39	44.379	.6520
2	53	56.162	.1780
3	52	47.382	.4501
4	37	29.981	1.6433
5	13	15.176	.3120
6	5)) 7	6.402)) 8.717	.3382
7	2)	2.315)	
TOTALS	220	219.331	3.6961 = χ^2

The degrees of freedom = d.f. = N - P - 1 = 7 - 1 - 1 = 5

From the χ^2 table, a level of significance of 50% = 4.351, and a level of significance of 70% = 3.000. Using linear interpolation, a value of χ^2 = 3.6961 and 5 degrees of freedom will yield a level of significance of 59.7%.

TABLE B - 5

THE FREQUENCY DISTRIBUTION OF CUSTOMER ARRIVALS, THE THEORETICAL POISSON DISTRIBUTION, AND THE CHI-SQUARE GOODNESS-OF-FIT TEST, FOR SAMPLE PERIOD 4: (Friday)

From the computer run:

Sample Period 4: The mean arrival rate = 1.926 customers per minute.
The variance = .0038 customers per minute.
The standard deviation = .0617 customers per minute.

Customer Arrivals Per One Minute Interval X	Frequency Observed F_o	Theoretical Poisson Frequency Calculated F_c	$\frac{(F_o - F_c)^2}{F_c} = \chi^2$
0	40	43.89	.3447
1	84	84.57	.0038
2	88	81.48	.5217
3	51	52.34	.0344
4	26	25.21	.0246
5	9)) 12	9.72)) 12.84	.0588
6	3)	3.12)	
TOTALS	301	300.33	.988 = χ^2

The degrees of freedom = d.f. = N - P - 1 = 6 - 1 - 1 = 4

From the χ^2 table, a level of significance of 90% = 1.064, and a level of significance of 95% = .711. Using linear interpolation, a value of χ^2 = .988 and 4 degrees of freedom will yield a level of significance of 91.1%.

TABLE B - 6

THE FREQUENCY DISTRIBUTION OF CUSTOMER ARRIVALS, THE THEORETICAL POISSON DISTRIBUTION, AND THE CHI-SQUARE GOODNESS-OF-FIT TEST, FOR SAMPLE PERIOD 5: (Saturday)

From the computer run:

Sample Period 5: The mean arrival rate = 2.668 customers per minute.
The variance = .0021 customers per minute.
The standard deviation = .0460 customers per minute.

Customer Arrivals Per One Minute Interval X	Frequency Observed F_o	Theoretical Poisson Frequency Calculated F_c	$\frac{(F_o - F_c)^2}{F_c} = \chi^2$
0	30	31.88	.1109
1	83	85.06	.0499
2	108	113.47	.2618
3	109	100.95	.6419
4	74	67.34	.6587
5	33	35.93	.2389
6	14	15.98	.2453
7	5) 7	6.09) 8.12	.1544
8	2)	2.03)	
TOTALS	458	458.7	2.3618 = χ^2

The degrees of freedom = d.f. = N - P - 1 = 8 - 1 - 1 = 6

From the χ^2 table, a level of significance of 80% = 3.070, and a level of significance of 90% = 2.204. Using linear interpolation, a χ^2 value of 2.3618 and 6 degrees of freedom will yield a level of significance of 88.2%.

TABLE B - 7

PERCENTAGE COMPOSITION OF THE OBSERVED FREQUENCY DISTRIBUTIONS, BY SAMPLE PERIOD

Customer Arrivals Per One Minute Interval	Monday, Tuesday, Wednesday		Thursday before 5 p.m.		Thursday after 5 p.m.		Friday		Saturday	
X	F_o	% of total	F_o	% of total	F_o	% of total	F_o	% of total	F_o	% of total
0	101	36.0	26	12.4	19	8.6	40	13.3	30	6.6
1	113	40.3	61	29.0	39	17.7	84	27.9	83	18.1
2	46	16.3	64	30.4	53	24.2	88	29.3	108	23.6
3	14	4.9	33	15.8	52	23.6	51	16.9	109	23.8
4	6	2.2	14	6.7	37	16.8	26	8.6	74	16.1
5	1	.3	8	3.8	13	5.9	9	3.0	33	7.2
6	-	-	2	.95	5	2.3	3	1.0	14	3.1
7	-	-	2	.95	2	.9	-	-	5	1.1
8	-	-	-	-	-	-	-	-	2	.4
TOTALS	281	100.0	210	100.0	220	100.0	301	100.0	458	100.0

TABLE B - 8

PERCENTAGE COMPOSITION OF THE CALCULATED THEORETICAL POISSON FREQUENCY DISTRIBUTIONS, BY SAMPLE PERIOD

Customer Arrivals Per One Minute Interval	Monday, Tuesday, Wednesday		Thursday before 5 p.m.		Thursday after 5 p.m.		Friday		Saturday	
X	F_o	% of total	F_o	% of total	F_o	% of total	F_o	% of total	F_o	% of total
0	105.54	37.6	29.841	14.2	17.534	8.0	43.89	14.6	31.88	6.9
1	103.43	36.8	58.261	27.7	44.379	20.2	84.57	28.2	85.06	18.6
2	50.68	18.0	56.874	26.9	56.162	25.6	81.48	27.1	113.47	24.8
3	16.52	5.8	37.013	17.6	47.382	21.6	52.34	17.4	100.95	22.0
4	4.05	1.5	18.066	8.6	29.981	13.7	25.21	8.4	67.34	14.7
5	.79	.3	7.054	3.4	15.176	6.9	9.72	3.2	35.93	7.8
6	-	-	2.295	1.2	6.402	2.9	3.12	1.1	15.98	3.5
7	-	-	.640	.3	2.315	1.1	-	-	6.09	1.3
8	-	-	-	-	-	-	-	-	2.03	.4
TOTALS	281.01	100.0	210.04	99.9	219.331	100.0	300.33	100.0	458.73	100.0

TABLE B - 9

THE FREQUENCY DISTRIBUTIONS OF CUSTOMER SERVICE TIMES FOR EACH SAMPLE PERIOD: CASHIER ALONE

Range (seconds)	Mid Value	Monday Tuesday Wednesday	Thursday (before 5 p.m.)	Thursday (after 5 p.m.)	Friday	Saturday	Totals
0- 14	7	-	-	-	-	-	-
15- 29	22	-	-	-	-	-	-
30- 44	37	3	2	-	2	-	7
45- 59	52	1	-	-	1	-	2
60- 74	67	-	-	-	-	-	-
75- 89	82	2	2	-	1	-	5
90-104	97	10	11	4	7	3	35
105-119	112	7	10	2	5	2	26
120-134	127	8	5	3	4	1	21
135-149	142	6	6	3	6	6	27
150-164	157	5	4	5	5	5	24
165-179	172	7	8	9	7	10	41
180-194	187	5	9	6	6	9	35
195-209	202	3	1	8	6	6	24
210-224	217	3	5	15	11	11	45
225-239	232	3	1	6	3	4	17
140-254	247	1	-	-	-	1	2
255-269	262	-	-	2	-	2	4
270-284	277	-	-	-	-	2	2
285-299	292	-	-	1	-	2	3
TOTALS		64	64	64	64	64	320

TABLE B - 10

THE MEAN, VARIANCE, AND STANDARD DEVIATION OF THE FREQUENCY DISTRIBUTIONS OF CUSTOMER SERVICE TIMES, AS CALCULATED BY THE COMPUTER FOR EACH SAMPLE PERIOD: CASHIER ALONE

Sample Period	Mean	Variance	Standard Deviation
	Minutes Per Customer		
Monday Tuesday Wednesday	2.3626	.07829	.2798
Thursday (before 5 p.m.)	2.3744	.01892	.1376
Thursday (after 5 p.m.)	3.1129	.05962	.2442
Friday	2.6598	.01058	.1029
Saturday	3.1558	.03958	.1989
All Sample Periods Together	2.7331	.15903	.3988

FIGURE B - 1

SERVICE TIME DISTRIBUTION FUNCTION S_o (t): THE PROBABILITY THAT THE SERVICE OPERATION TAKES LONGER THAN TIME t: SAMPLE PERIOD 1: MONDAY, TUESDAY, WEDNESDAY: CASHIER ALONE

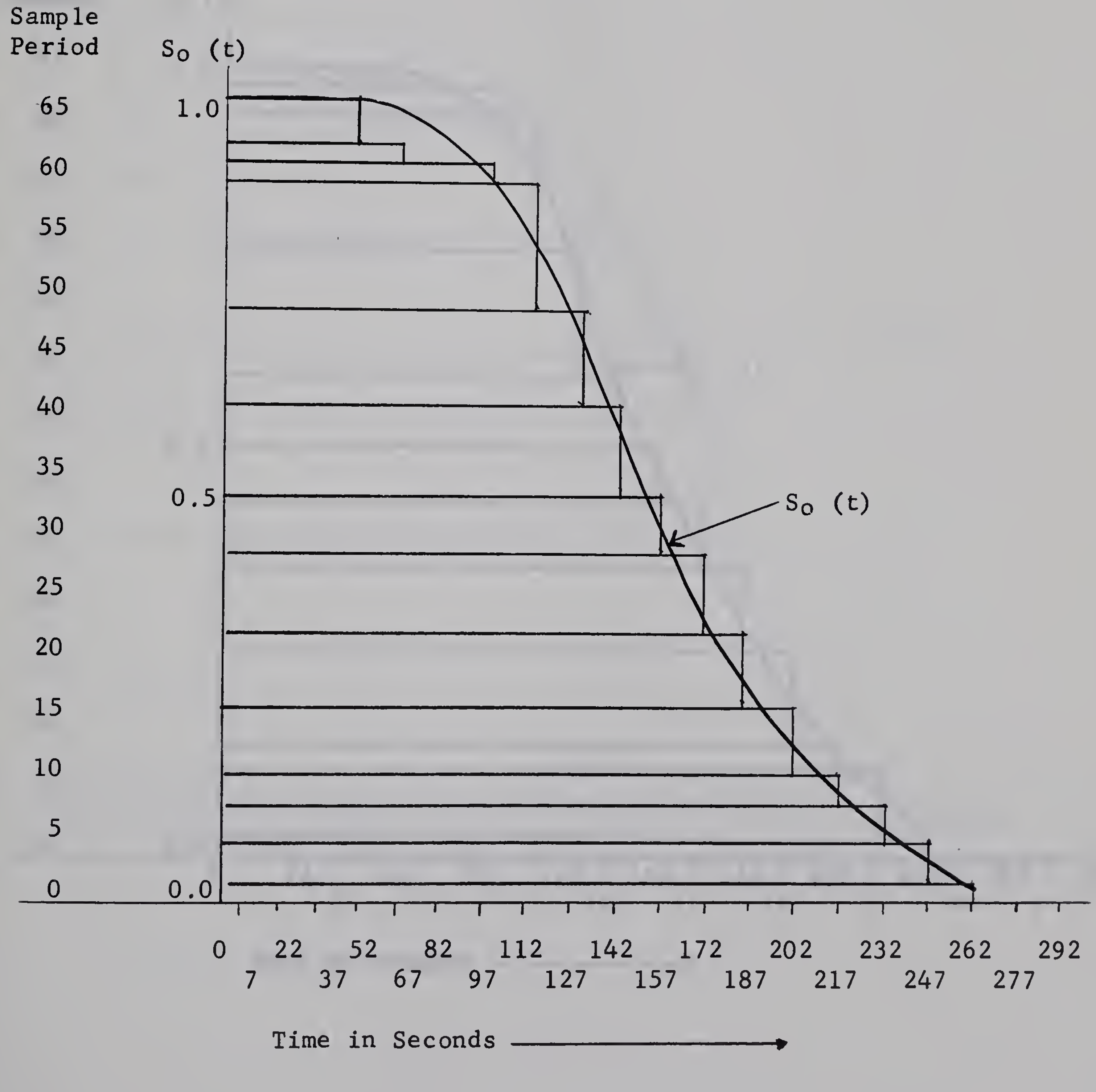


FIGURE B - 2

SERVICE TIME DISTRIBUTION FUNCTION S_o (t); THE PROBABILITY THAT THE SERVICE OPERATION TAKES LONGER THAN TIME t: SAMPLE PERIOD 2: THURSDAY BEFORE 5 P.M.: CASHIER ALONE

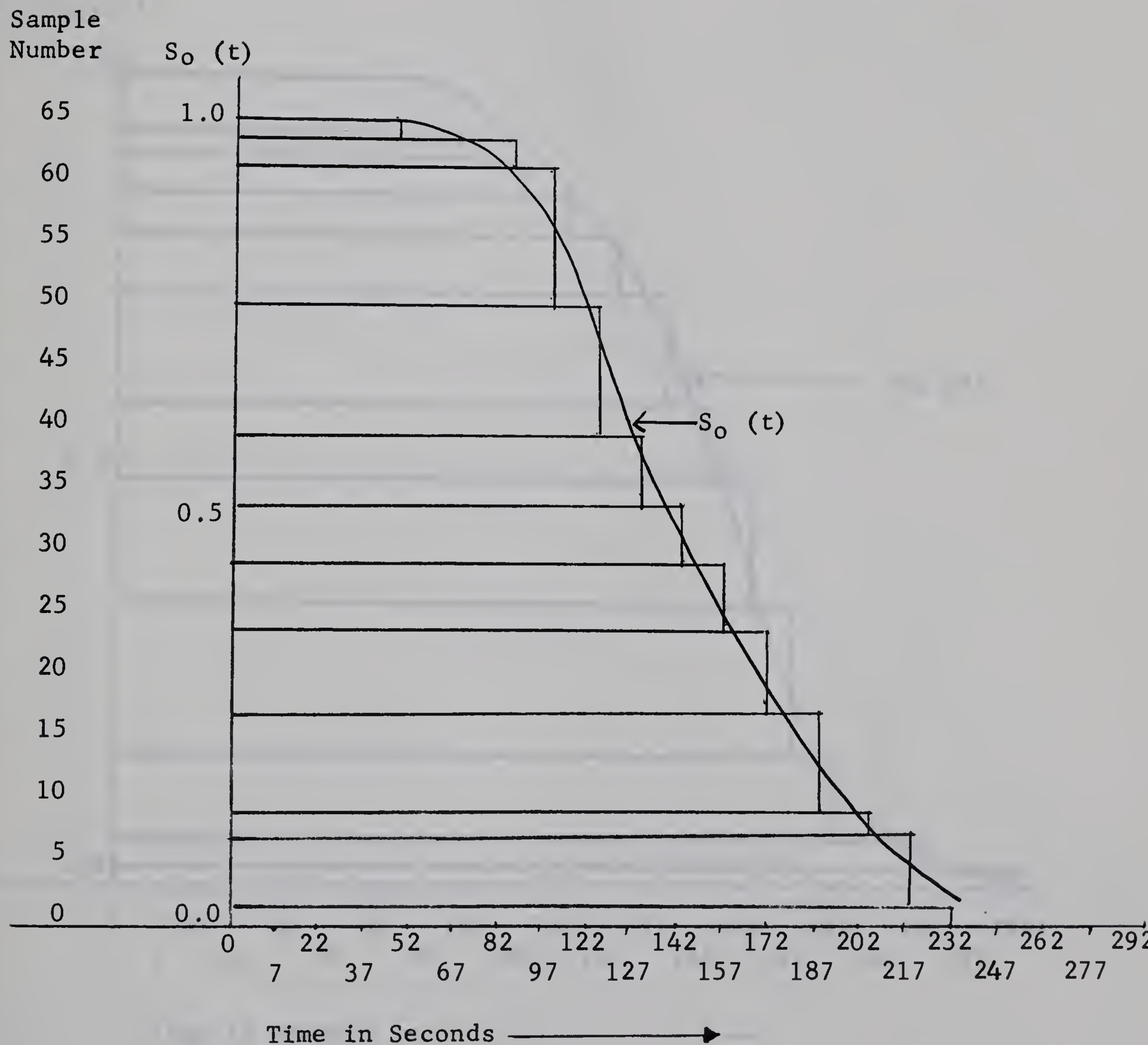


FIGURE B - 3

SERVICE TIME DISTRIBUTION FUNCTION S_o (t); THE PROBABILITY THAT THE SERVICE OPERATION TAKES LONGER THAN TIME t: SAMPLE PERIOD 3: THURSDAY AFTER 5 P.M.: CASHIER ALONE

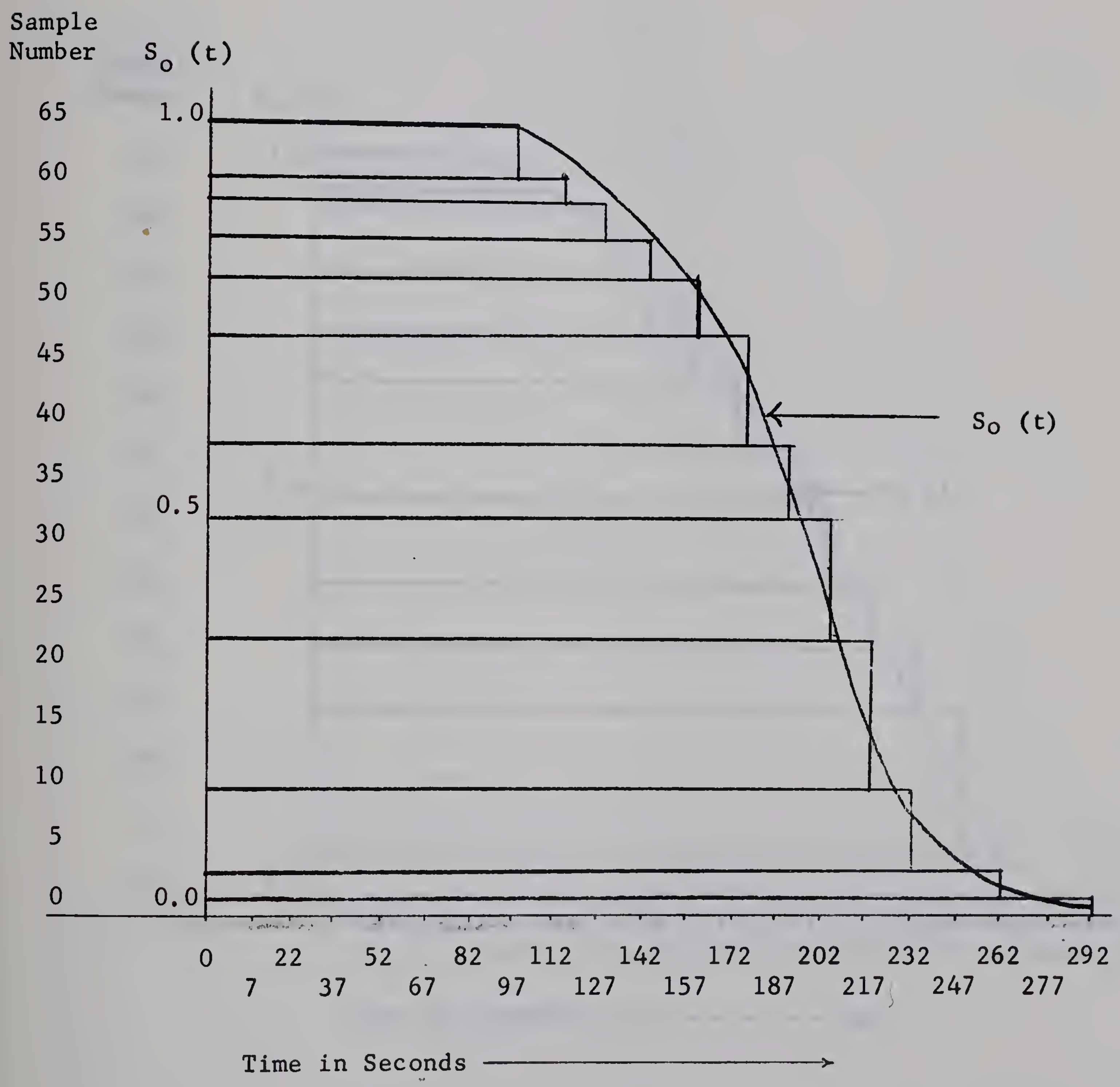


FIGURE B - 4

SERVICE TIME DISTRIBUTION FUNCTION $S_o(t)$: THE PROBABILITY THAT THE SERVICE OPERATION TAKES LONGER THAN TIME t: SAMPLE PERIOD 4: FRIDAY: CASHIER ALONE

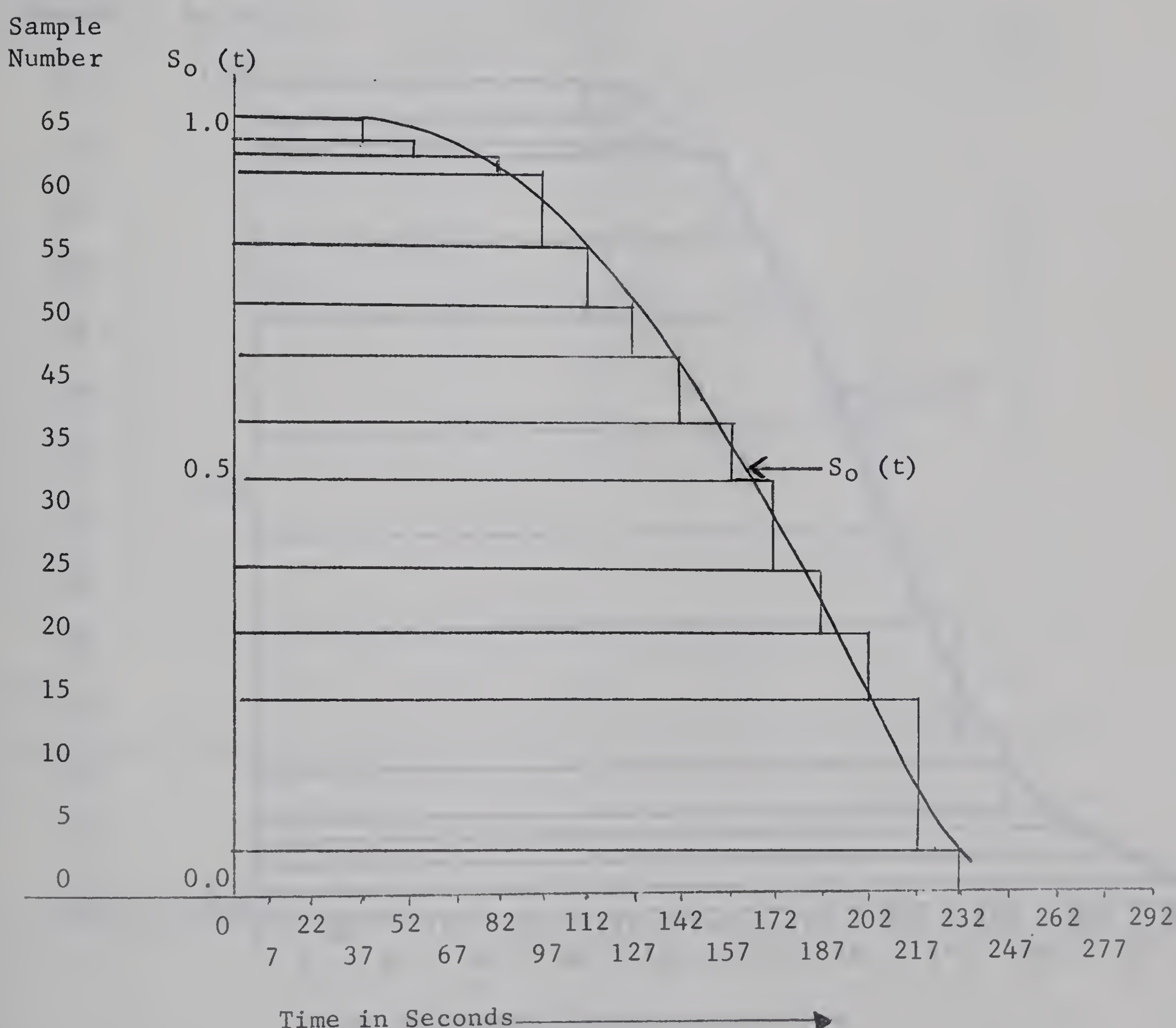


FIGURE B - 5

SERVICE TIME DISTRIBUTION FUNCTION S_o (t): THE PROBABILITY THAT THE SERVICE OPERATION TAKES LONGER THAN TIME t: SAMPLE PERIOD 5: SATURDAY: CASHIER ALONE

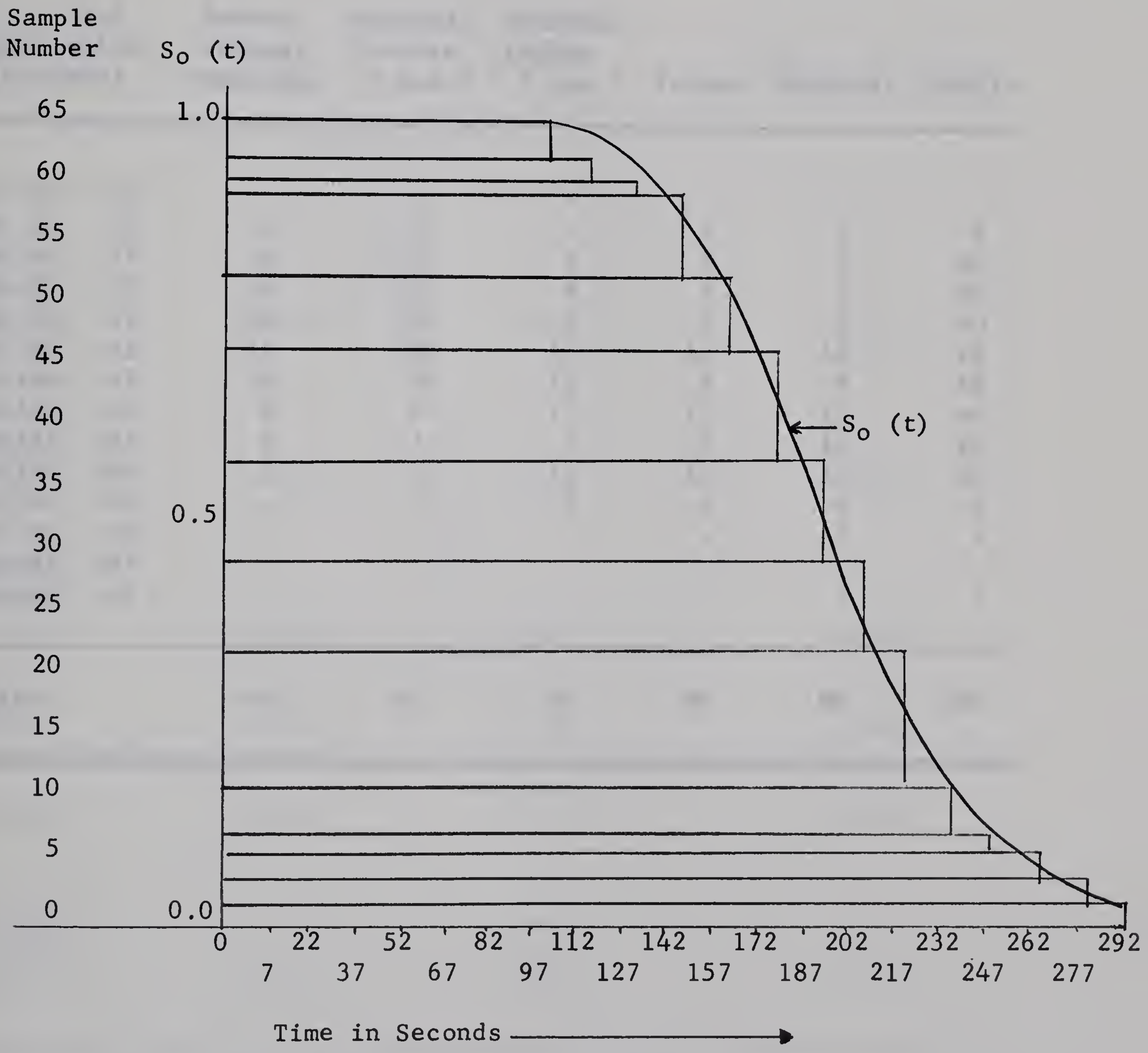


TABLE B - 11

THE FREQUENCY DISTRIBUTIONS OF CUSTOMER SERVICE TIMES FOR EACH SAMPLE PERIOD: CASHIER AND BAG BOY

Range (seconds)	Mid Value	Monday Tuesday Wednesday	Thursday (before 5 p.m.)	Thursday (after 5 p.m.)	Friday	Saturday	Totals
0- 14	7	-	-	-	-	-	-
15- 29	22	3	1	-	1	1	6
30- 44	37	10	12	5	5	4	36
45- 59	52	18	17	8	9	4	56
60- 74	67	16	12	2	9	1	40
75- 89	82	14	18	14	16	14	76
90-104	97	9	8	13	9	9	48
105-119	112	8	9	15	11	13	56
120-134	127	1	1	7	7	10	26
135-149	142	1	2	13	12	17	45
150-164	157	-	-	2	1	4	7
165-179	172	-	-	1	-	2	3
180-194	187	-	-	-	-	-	-
195-209	202	-	-	-	-	1	1
TOTALS		80	80	80	80	80	400

TABLE B - 12

THE MEAN, VARIANCE, AND STANDARD DEVIATION OF THE FREQUENCY DISTRIBUTIONS OF CUSTOMER SERVICE TIMES, AS CALCULATED BY THE COMPUTER FOR EACH SAMPLE PERIOD: CASHIER AND BAG BOY

Sample Period	Mean	Variance	Standard Deviation
	Minutes Per Customer		
Monday Tuesday Wednesday	1.1733	.00424	.06512
Thursday (before 5 p.m.)	1.2139	.00758	.08705
Thursday (after 5 p.m.)	1.6764	.01481	.12170
Friday	1.5514	.01052	.10259
Saturday	1.8295	.01268	.11260
All Sample Periods Together	1.4889	.07670	.27696

FIGURE B - 6

SERVICE TIME DISTRIBUTION FUNCTION S_o (t): THE PROBABILITY THAT THE SERVICE OPERATION TAKES LONGER THAN TIME t: SAMPLE PERIOD 1: MONDAY, TUESDAY, WEDNESDAY: CASHIER AND BAG BOY

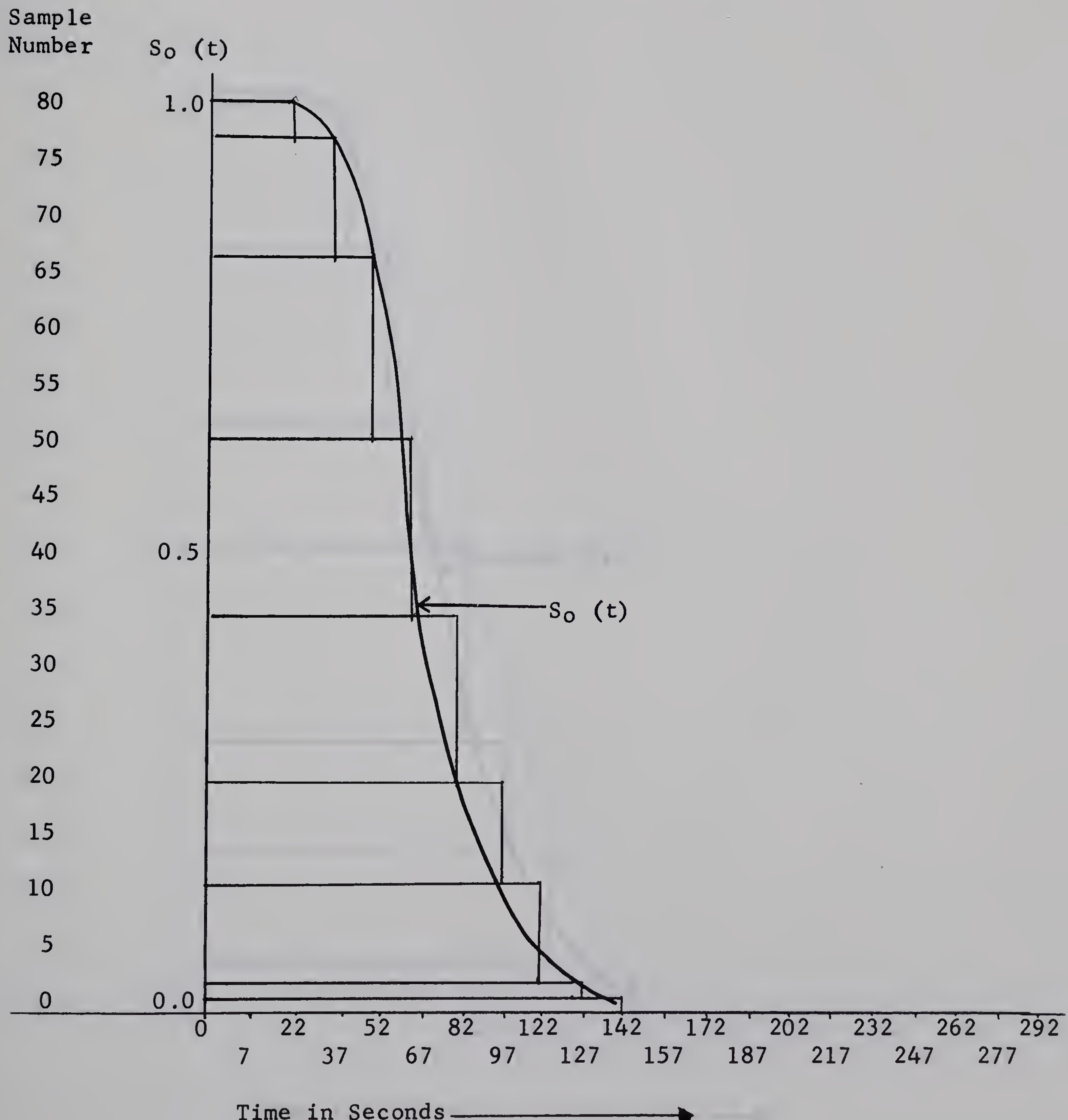


FIGURE B - 7

SERVICE TIME DISTRIBUTION FUNCTION S_o (t): THE PROBABILITY THAT THE SERVICE OPERATION TAKES LONGER THAN TIME t: SAMPLE PERIOD 2: THURSDAY BEFORE 5 P.M.: CASHIER AND BAG BOY

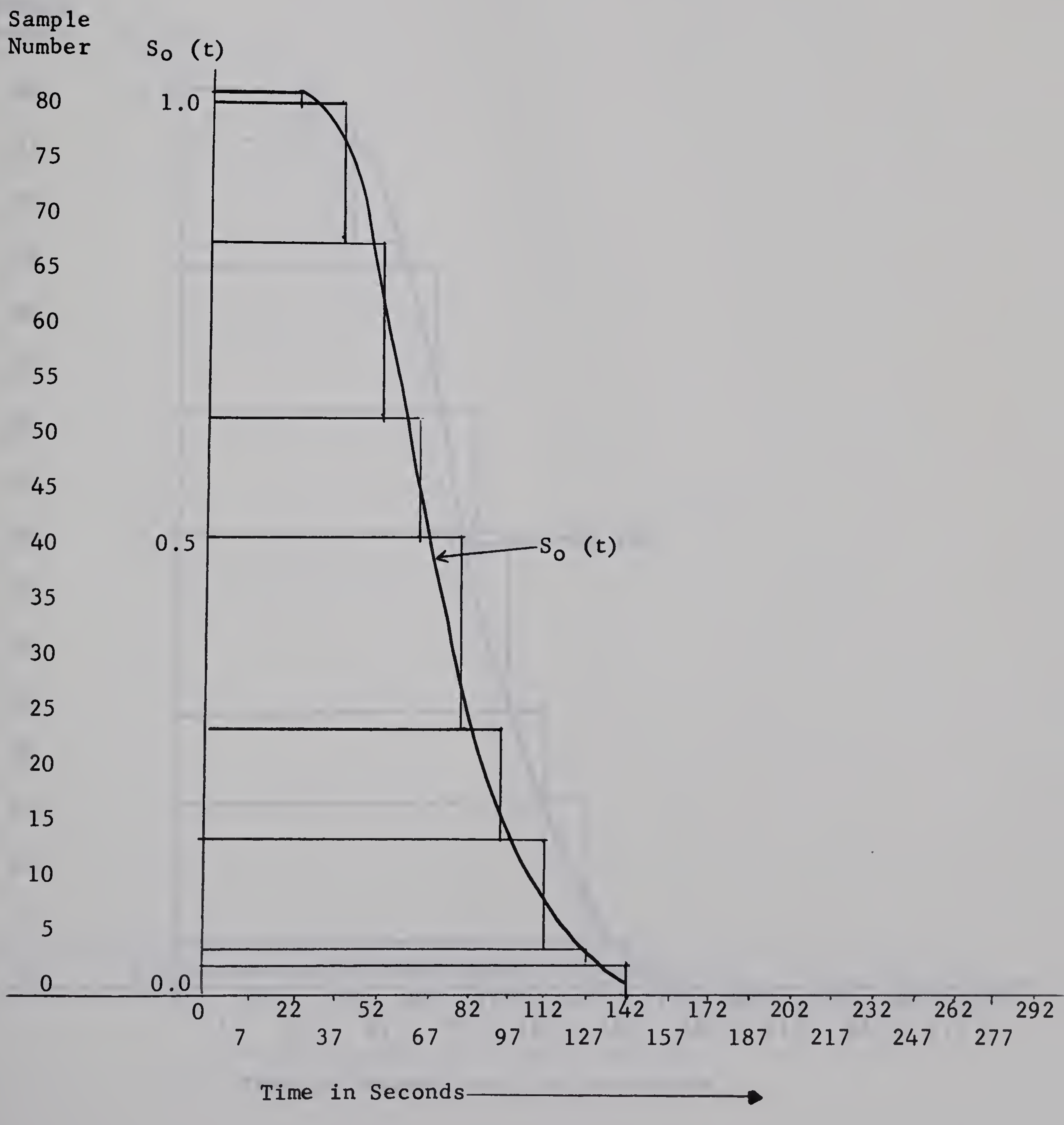


FIGURE B - 8

SERVICE TIME DISTRIBUTION FUNCTION $S_o(t)$: THE PROBABILITY THAT THE SERVICE OPERATION TAKES LONGER THAN TIME t: SAMPLE PERIOD 3: THURSDAY AFTER 5 P.M.: CASHIER AND BAG BOY

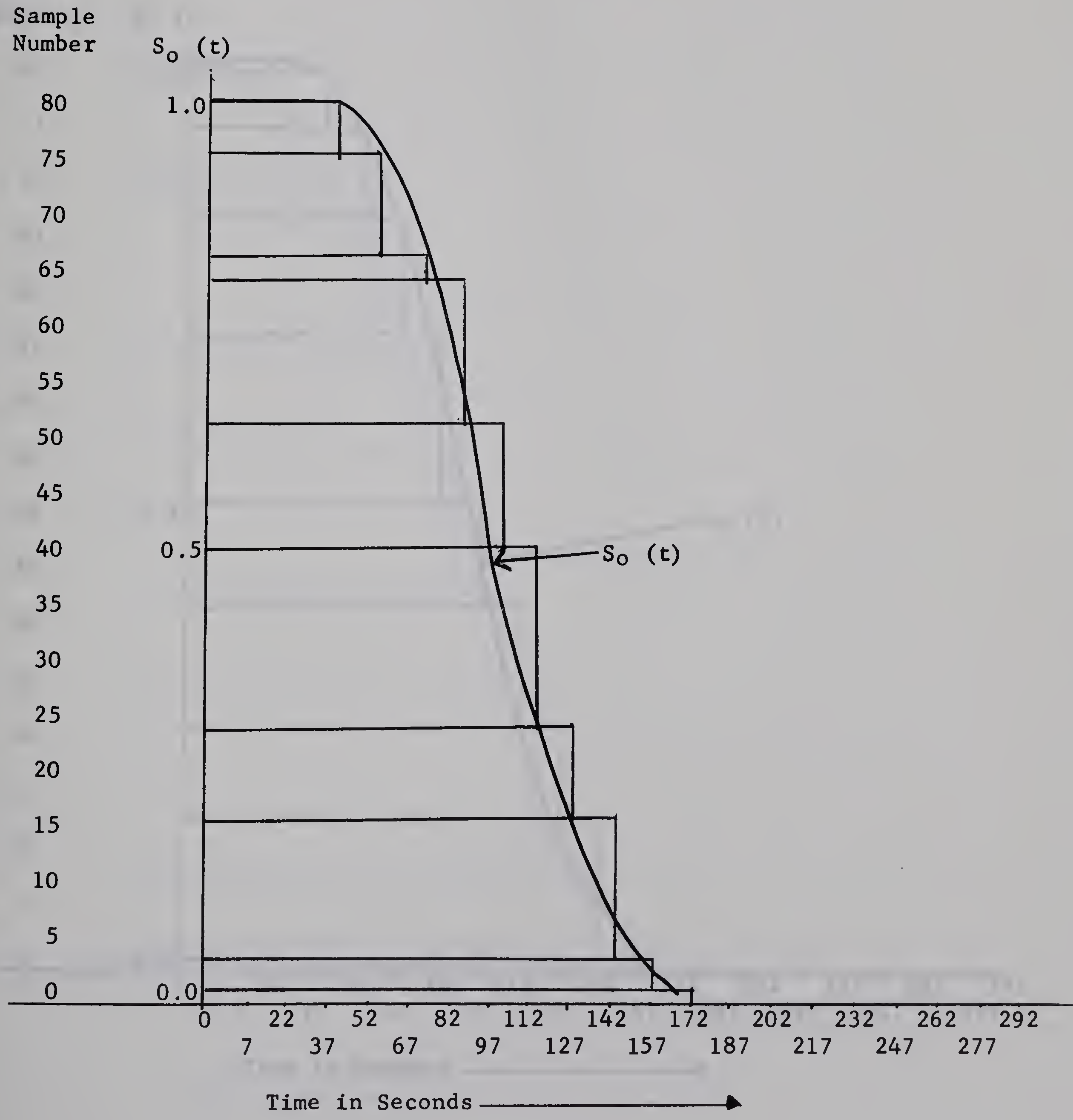
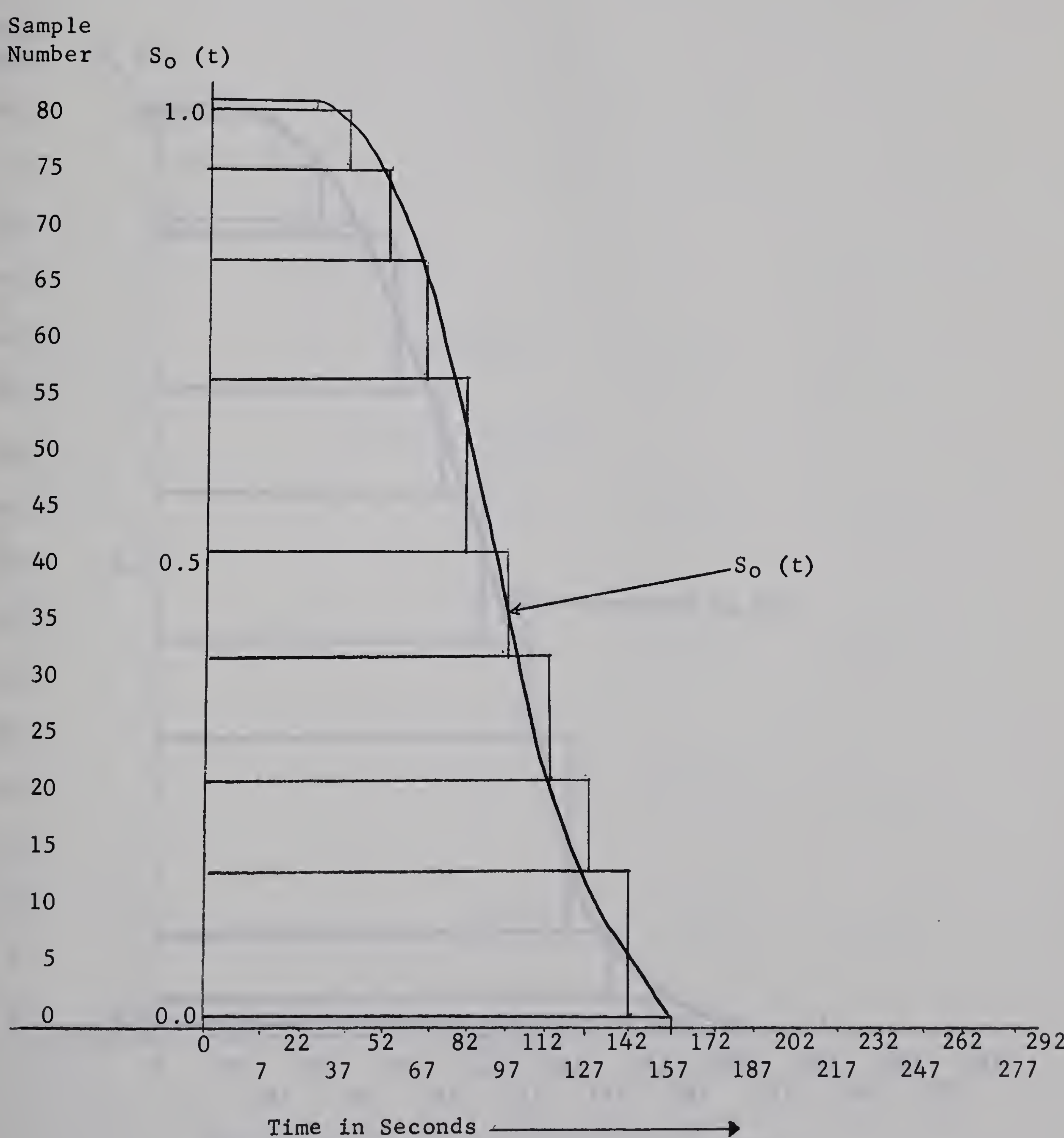


FIGURE B - 9

SERVICE TIME DISTRIBUTION FUNCTION S_o (t): THE PROBABILITY THAT THE SERVICE OPERATION TAKES LONGER THAN TIME t: SAMPLE PERIOD 4: FRIDAY CASHIER AND BAG BOY



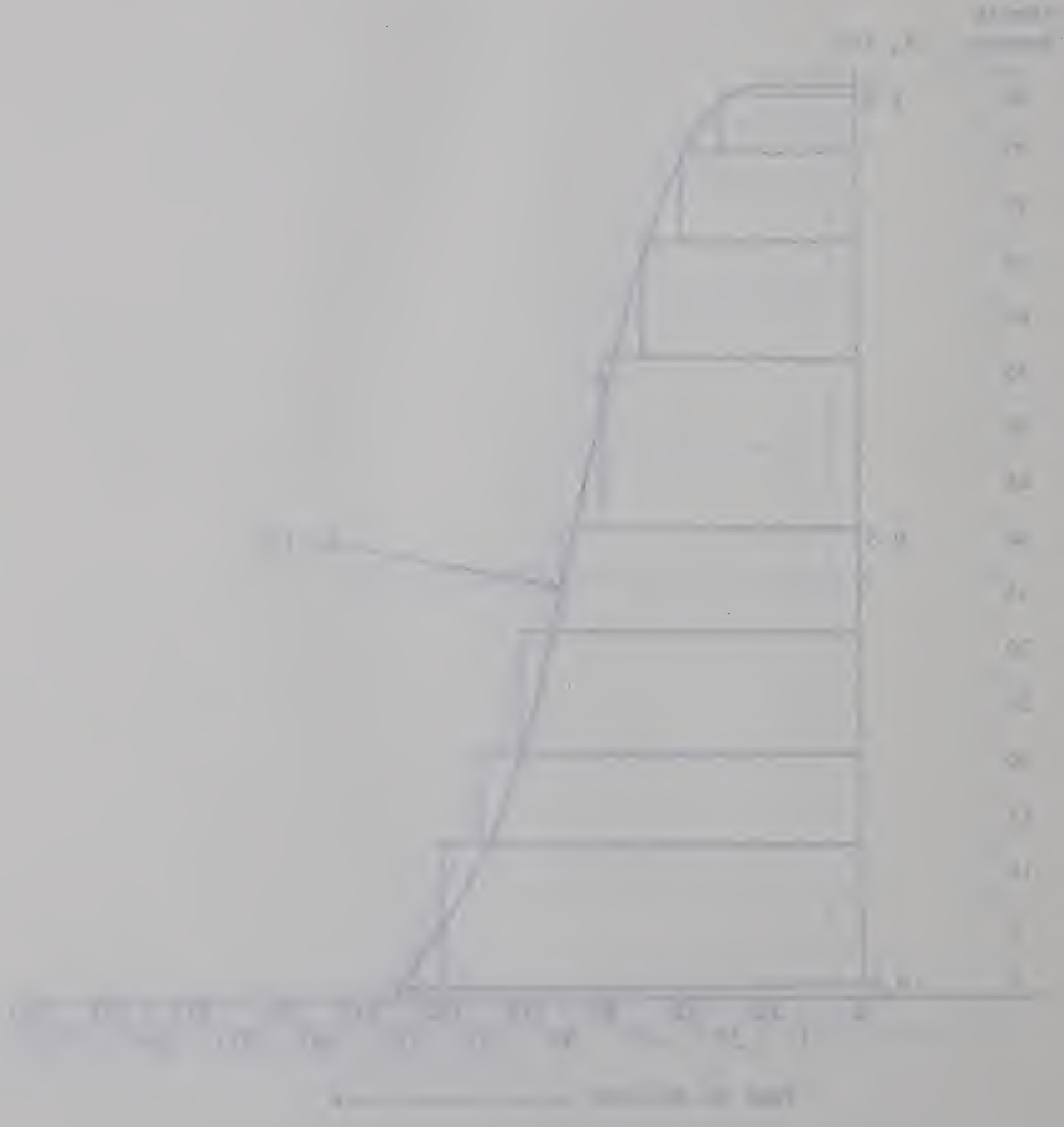
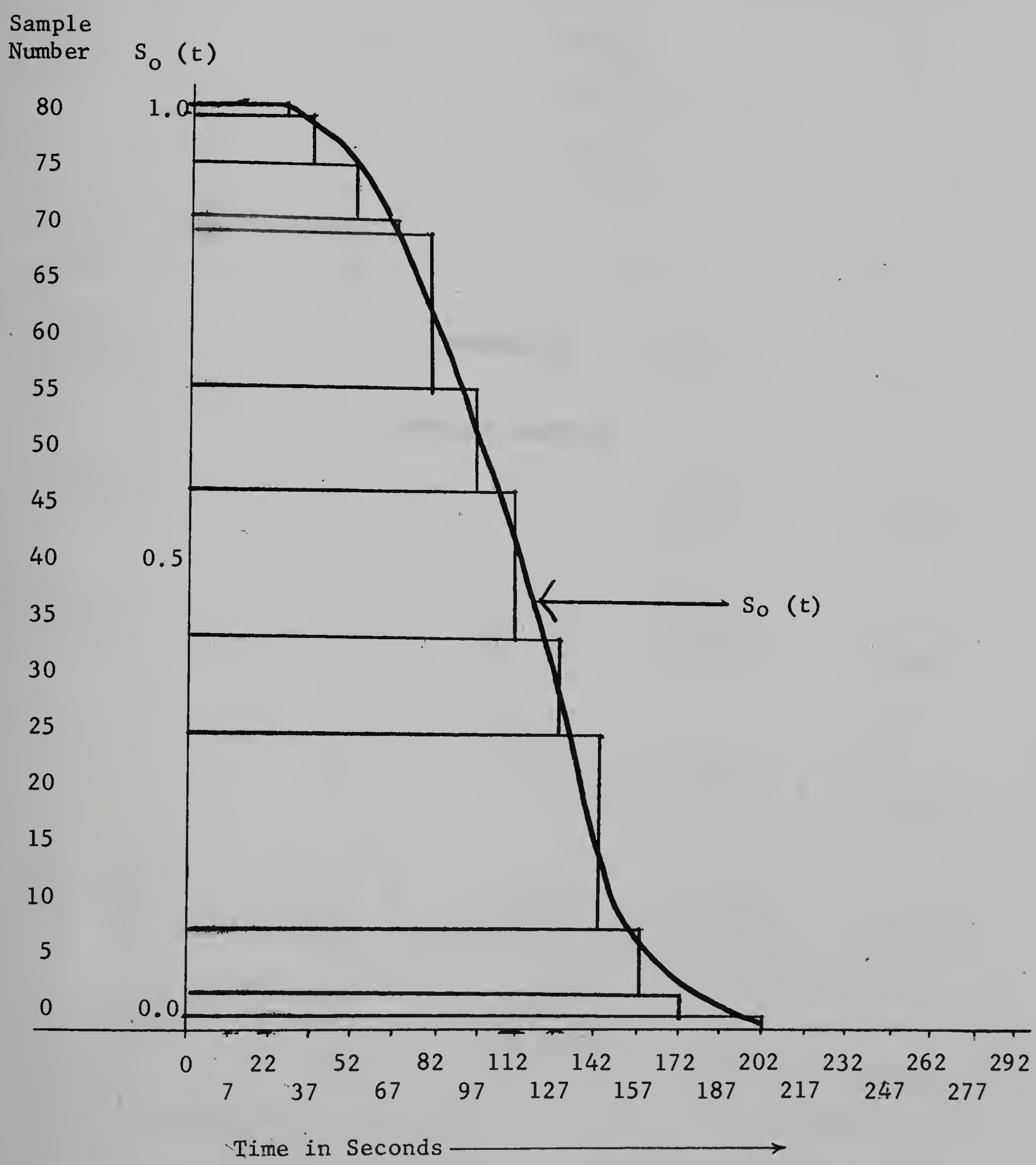


FIGURE B - 10

SERVICE TIME DISTRIBUTION FUNCTION S_o (t); THE PROBABILITY THAT THE SERVICE OPERATION TAKES LONGER THAN TIME t: SAMPLE PERIOD 5: SATURDAY: CASHIER AND BAG BOY



APPENDIX C

COMPUTER RESULTS

TABLE C - 1

THE SIMPLE CORRELATION COEFFICIENTS BETWEEN THE THREE VARIABLES; SIZE OR ORDER, AVERAGE ARRIVAL RATE, AND AVERAGE SERVICE TIME, AS CALCULATED BY THE COMPUTER FOR EACH SAMPLE PERIOD: CASHIER ALONE

Variables: 1. = The average size of order.

2. = The average arrival rate per minute.

3. = The average service time per customer.

Sample Period	Variables	1.	2.	3.
Monday	1.	1.000000		
Tuesday	2.	.964582	1.000000	
Wednesday	3.	.930645	.919831	1.000000
Thursday	1.	1.0000000		
(before	2.	.9380191	1.0000000	
5 p.m.)	3.	.9139045	.9871842	1.000000
Thursday	1.	1.0000000		
(after	2.	.9312234	1.0000000	
5 p.m.)	3.	.9084387	.8619314	1.000000
Friday	1.	1.0000000		
	2.	.9442940	1.0000000	
	3.	.9643925	.9541738	1.000000
Saturday	1.	1.0000000		
	2.	.9532977	1.0000000	
	3.	.9200798	.9774875	1.000000
All Sample	1.	1.0000000		
Periods	2.	.8790850	1.0000000	
Together	3.	.8311351	.8078692	1.000000

TABLE C - 2

THE SIMPLE CORRELATION COEFFICIENTS BETWEEN THE THREE VARIABLES; SIZE OF ORDER, AVERAGE ARRIVAL RATE, AND AVERAGE SERVICE TIME, AS CALCULATED BY THE COMPUTER FOR EACH SAMPLE PERIOD: CASHIER AND BAG BOY

Variables: 1. = The average size of order.

2. = The average arrival rate per minute.

3. = The average service time per customer.

Sample Period	Variables	1.	2.	3.
Monday	1.	1.000000		
Tuesday	2.	.964582	1.000000	
Wednesday	3.	.874629	.821534	1.000000
Thursday	1.	1.000000		
(before	2.	.938019	1.000000	
5 p.m.)	3.	.894347	.934130	1.000000
Thursday	1.	1.000000		
(after	2.	.931223	1.000000	
5 p.m.)	3.	.902007	.907920	1.000000
Friday	1.	1.000000		
	2.	.944294	1.000000	
	3.	.928777	.943093	1.000000
Saturday	1.	1.000000		
	2.	.953298	1.000000	
	3.	.853802	.943093	1.000000
All Sample	1.	1.000000		
Periods	2.	.879085	1.000000	
Together	3.	.861001	.839865	1.000000

TABLE C - 3

COMPUTER OUTPUT FOR SAMPLE PERIOD 1:
MONDAY, TUESDAY, WEDNESDAY: NO BAG BOY

REGRESSION COEFFICIENTS

VARIABLE	COEFFICIENT	STANDARD ERROR	T-VALUE	VARIANCE (PERCENT)
C	-9.2188632 E-01			
1	9.4382509 E-01	1.5150290 E-01	6.23	86.61
TOTAL VARIANCE				86.61

ANALYSIS OF VARIANCE TABLE

SOURCE	D.F.	SUM OF SQUARES	MEAN SQUARE	F-RATIO
REGRESSION	1	4.7462132 E-01	4.7462132 E-01	38.81
ERROR	6	7.3376559 E-02	1.2229426 E-02	
TOTAL	7	5.4799788 E-01		

STANDARD ERROR OF ESTIMATE 1.1058674 E-01

TABLE C - 3 (Continued)

COMPUTER OUTPUT FOR SAMPLE PERIOD 1:
MONDAY, TUESDAY, WEDNESDAY: NO BAG BOY

REGRESSION COEFFICIENTS

VARIABLE	COEFFICIENT	STANDARD ERROR	T-VALUE	VARIANCE (PERCENT)
C	-6.1292931 E-01			
1	6.3245884 E-01	6.1237790 E-01	1.03	86.61
2	7.9080910 E-01	1.5002330 E 00	0.53	0.70
TOTAL VARIANCE				87.31

ANALYSIS OF VARIANCE TABLE

SOURCE	D.F.	SUM OF SQUARES	MEAN SQUARE	F-RATIO
REGRESSION	2	4.7848433 E-01	2.3924216 E-01	17.21
ERROR	5	6.9513553 E-02	1.3902710 E-02	
TOTAL	7	5.4799788 E-01		

STANDARD ERROR OF ESTIMATE 1.1790975 E-01

EFFECT OF LAST VARIABLE
CONTRIBUTION TO SUM OF SQUARES 3.8630059 E-03
RATIO OF CONTRIBUTION TO ERROR MEAN SQUARE 0.28

DETERMINANT OF CORRELATION MATRIX 6.9581897 E-02

TABLE C - 3 (Continued)

COMPUTER OUTPUT FOR SAMPLE PERIOD 1:
MONDAY, TUESDAY, WEDNESDAY: NO BAG BOY

CBSERVEC ANC ESTIMATEC VALUES AND RESIDUALS

NUMBER	CBSERVEC VALUE	ESTIMATED VALLE	RESIDUAL
1	2.523CCCC E CC	2.5543604 E 00	-3.1360395 E-02
2	2.554C000 E CC	2.5164035 E 00	3.7596533 E-02
3	2.117CCOC E CC	2.1874966 E 00	-7.0496569 E-02
4	2.710C000 E CC	2.7472839 E 00	-3.7283903 E-02
5	2.617COOC E CC	2.4294333 E 00	1.8756671 E-01
6	1.960CC00 E CC	1.896541C E 00	6.3458962 E-02
7	2.335C000 E CC	2.3377126 E 00	-2.7126303 E-03
8	2.085C000 E OC	2.2317687 E 00	-1.4676871 E-01

SUM CF RESICUALS	-3.9968028 E-15
SUM CF SQUARES CF RESICUALS	6.9513553 E-02
MEAN OF RESIUCALS	-4.9960036 E-16
STANCARC CEVIATICN OF RESICUALS	9.9651931 E-02
CURBIN-WATSCN STATISTIC	1.5627690 E 00
CCRRELATION CF RESICUALS	-0.02099 -0.44866
	0.10016 -0.44892
NUMBER CF RUNS	5
PCSITIVE SIGNS	3
NEGATIVE SIGNS	5
MEAN	4.7500000 E 00
STANCARC CEVIATICN	1.2137604 E 00

AC,MC UNCERFLCW AT 022273

AC,MC UNCERFLCW AT 022315

MC UNCERFLCW AT 022326

CCEFFICIENT CF SKEWNESS	6.3767079 E-01
STANCARC CEVIATICN	7.5210144 E-01
CCEFFICIENT CF KURTOSIS	1.1757576 E 00
STANCARC CEVIATICN	1.4808805 E 00

TABLE C - 4

COMPUTER OUTPUT FOR SAMPLE PERIOD 1:
MONDAY, TUESDAY, WEDNESDAY: WITH BAG BOY

REGRESSION COEFFICIENTS

VARIABLE	COEFFICIENT	STANDARD ERROR	T-VALUE	VARIANCE (PERCENT)
C	4.5478154 E-01			
1	2.0645645 E-01	4.6718153 E-02	4.42	76.50
TOTAL VARIANCE				76.50

ANALYSIS OF VARIANCE TABLE

SOURCE	D.F.	SUM OF SQUARES	MEAN SQUARE	F-RATIO
REGRESSION	1	2.2710209 E-02	2.2710209 E-02	19.53
ERROR	6	6.9772901 E-03	1.1628817 E-03	
TOTAL	7	2.9687499 E-02		

STANDARD ERROR OF ESTIMATE 3.4101051 E-02

TABLE C - 4 (Continued)

COMPUTER OUTPUT FOR SAMPLE PERIOD 1:
MONDAY, TUESDAY, WEDNESDAY: WITH BAG BOY

REGRESSION COEFFICIENTS

VARIABLE	COEFFICIENT	STANDARD ERROR	T-VALUE	VARIANCE (PERCENT)
O	3.8296798 E-01			
1	2.7883002 E-01	1.9108807 E-01	1.46	76.50
2	-1.8381463 E-01	4.6813679 E-01	-0.39	0.70
TOTAL VARIANCE				77.20

ANALYSIS OF VARIANCE TABLE

SOURCE	D.F.	SUM OF SQUARES	MEAN SQUARE	F-RATIO
REGRESSION	2	2.2918918 E-02	1.1459459 E-02	8.47
ERROR	5	6.7685806 E-03	1.3537161 E-03	
TOTAL	7	2.9687499 E-02		

STANDARD ERROR OF ESTIMATE 3.6792881 E-02

EFFECT OF LAST VARIABLE
CONTRIBUTION TO SUM OF SQUARES 2.0870949 E-04
RATIO OF CONTRIBUTION TO ERROR MEAN SQUARE 0.15

DETERMINANT OF CORRELATION MATRIX 6.9581897 E-02

TABLE C - 4 (Continued)

COMPUTER OUTPUT FOR SAMPLE PERIOD 1:
MONDAY, TUESDAY, WENDESDAY: WITH BAG BOY

OBSERVED AND ESTIMATED VALUES AND RESIDUALS

NUMBER	OBSERVED VALUE	ESTIMATED VALUE	RESIDUAL
1	1.1670000 E 00	1.2095925 E 00	-4.2592479 E-02
2	1.2420000 E 00	1.2141567 E 00	2.7843237 E-02
3	1.1420000 E 00	1.1330474 E 00	8.9525937 E-03
4	1.2420000 E 00	1.2414004 E 00	5.9958316 E-04
5	1.2420000 E 00	1.1917882 E 00	5.0211798 E-02
6	1.0670000 E 00	1.0601502 E 00	6.8497706 E-03
7	1.1670000 E 00	1.1832990 E 00	-1.6298984 E-02
8	1.1170000 E 00	1.1525655 E 00	-3.5565519 E-02

SUM OF RESIDUALS	-0.
SUM OF SQUARES OF RESIDUALS	6.7685806 E-03
MEAN OF RESIDUALS	-0.
STANDARD DEVIATION OF RESIDUALS	3.1095660 E-02
DURBIN-WATSON STATISTIC	1.5714551 E 00
CORRELATION OF RESIDUALS	-0.37586 -0.24705
	0.27191 -0.16243
NUMBER OF RUNS	3
POSITIVE SIGNS	5
NEGATIVE SIGNS	3
MEAN	4.7500000 E 00
STANDARD DEVIATION	1.2137604 E 00
COEFFICIENT OF SKEWNESS	1.4339034 E-01
STANDARD DEVIATION	7.5210144 E-01
COEFFICIENT OF KURTOSIS	-5.2604563 E-01
STANDARD DEVIATION	1.4808805 E 00

TABLE C - 5

COMPUTER OUTPUT FOR SAMPLE PERIOD 2:
THURSDAY BEFORE 5 p.m.: NO BAG BOY

REGRESSICN CCEFFICIENTS

VARIABLE	CCEFFICIENT	STANCARD ERRCR	T-VALUE	VARIANCE (PERCENT)
C	2.4C26438 E-C1			
2	1.C932244 E CC	7.2148465 E-C2	15.15	97.45
TCTAL VARIANCE				97.45

ANALYSIS OF VARIANCE TABLE

SCURCE	C.F.	SUM CF SQUARES	MEAN SQUARE	F-RATIC
REGRESSICN	1	1.2906894 E-C1	1.2906894 E-C1	229.60
ERRCR	6	3.3729398 E-C3	5.6215663 E-C4	
TCTAL	7	1.3244188 E-C1		

STANCARC ERRCR CF ESTIMATE 2.3709842 E-C2

TABLE C - 5 (Continued)

COMPUTER OUTPUT FOR SAMPLE PERIOD 2:
THURSDAY BEFORE 5 p.m.: NO BAG BOY

REGRESSION COEFFICIENTS

VARIABLE	COEFFICIENT	STANDARD ERROR	T-VALUE	VARIANCE (PERCENT)
C	1.4874921 E-01			
2	1.1978038 E 00	2.2252178 E-01	5.38	97.45
1	-1.9115315 E-02	3.8152174 E-02	-0.50	0.12
TOTAL VARIANCE				97.58

ANALYSIS OF VARIANCE TABLE

SOURCE	D.F.	SUM OF SQUARES	MEAN SQUARE	F-RATIO
REGRESSION	2	1.2923019 E-01	6.4615094 E-02	100.59
ERROR	5	3.2116939 E-03	6.4233879 E-04	
TOTAL	7	1.3244188 E-01		

STANDARD ERROR OF ESTIMATE 2.5344403 E-02

EFFECT OF LAST VARIABLE
CONTRIBUTION TO SUM OF SQUARES 1.6124583 E-04
RATIO OF CONTRIBUTION TO ERROR MEAN SQUARE 0.25

DETERMINANT OF CORRELATION MATRIX 1.2012017 E-01

TABLE C - 5 (Continued)

COMPUTER OUTPUT FOR SAMPLE PERIOD 2:
THURSDAY BEFORE 5 p.m.: NO BAG BOY

```
     OBSERVED AND ESTIMATED VALUES AND RESIDUALS

NUMBER      OBSERVED          ESTIMATED           RESIDUAL
              VALUE             VALUE
   1     2.6480000 E 00   2.6296584 E 00   1.8341636 E-02
   2     2.3040000 E 00   2.2975888 E 00   6.4111495 E-03
   3     2.3350000 E 00   2.3047248 E 00   3.0275217 E-02
   4     2.3670000 E 00   2.3800449 E 00  -1.3044806 E-02
   5     2.4600000 E 00   2.4679301 E 00  -7.9300603 E-03
   6     2.3040000 E 00   2.2896883 E 00   1.4311695 E-02
   7     2.3980000 E 00   2.4346350 E 00  -3.6634942 E-02
   8     2.1790000 E 00   2.1907299 E 00  -1.1729890 E-02

     SUM OF RESIDUALS                         -2.2204460 E-15
     SUM OF SQUARES OF RESIDUALS               3.2116939 E-03
     MEAN OF RESIDUALS                        -2.7755575 E-16
     STANDARD DEVIATION OF RESIDUALS           2.1419930 E-02
     DURBIN-WATSON STATISTIC                   1.9694069 E 00
     CORRELATION OF RESIDUALS                 -0.29478   0.19169
                                              -0.12183   0.16424

     NUMBER OF RUNS                              4
           POSITIVE SIGNS                        4
           NEGATIVE SIGNS                        4
           MEAN                                5.0000000 E 00
           STANDARD DEVIATION                  1.3093073 E 00

   MQ UNDERFLOW AT       022263

   MQ UNDERFLOW AT       022270

AC,MQ UNDERFLOW AT       022273

   MQ UNDERFLOW AT       022312

AC,MQ UNDERFLOW AT       022315

   MQ UNDERFLOW AT       022326
      COEFFICIENT OF SKEWNESS                 -3.1757711 E-01
             STANDARD DEVIATION                7.5210144 E-01
      COEFFICIENT OF KURTOSIS                 -2.5807787 E-01
             STANDARD DEVIATION                1.4808805 E 00
```

TABLE C - 6

COMPUTER OUTPUT FOR SAMPLE PERIOD 2:
THURSDAY BEFORE 5 p.m.: WITH BAG BOY

REGRESSION COEFFICIENTS

VARIABLE	COEFFICIENT	STANDARD ERROR	T-VALUE	VARIANCE (PERCENT)
C	-6.4161015 E-02			
2	6.5468964 E-01	1.0212702 E-01	6.41	87.26
TOTAL VARIANCE				87.26

ANALYSIS OF VARIANCE TABLE

SOURCE	D.F.	SUM OF SQUARES	MEAN SQUARE	F-RATIO
REGRESSION	1	4.6288604 E-02	4.6288604 E-02	41.10
ERROR	6	6.7582723 E-03	1.1263787 E-03	
TOTAL	7	5.3046876 E-02		

STANDARD ERROR OF ESTIMATE 3.3561566 E-02

TABLE C - 6 (Continued)

COMPUTER OUTPUT FOR SAMPLE PERIOD 2:
THURSDAY BEFORE 5 p.m.: WITH BAG BOY

REGRESSION COEFFICIENTS

VARIABLE	COEFFICIENT	STANDARD ERROR	T-VALUE	VARIANCE (PERCENT)
C	2.2601148 E-02			
2	5.5554171 E-01	3.1931249 E-01	1.74	87.26
1	1.8122526 E-02	5.4747295 E-02	0.33	0.27
TOTAL VARIANCE				87.53

ANALYSIS OF VARIANCE TABLE

SOURCE	D.F.	SUM OF SQUARES	MEAN SQUARE	F-RATIO
REGRESSION	2	4.6433536 E-02	2.3216768 E-02	17.55
ERROR	5	6.6133408 E-03	1.3226682 E-03	
TOTAL	7	5.3046876 E-02		

STANDARD ERROR OF ESTIMATE 3.6368505 E-02

EFFECT OF LAST VARIABLE
CONTRIBUTION TO SUM OF SQUARES 1.4493158 E-04
RATIO OF CONTRIBUTION TO ERROR MEAN SQUARE 0.11

DETERMINANT OF CORRELATION MATRIX 1.2012017 E-01

TABLE C - 6 (Continued)

COMPUTER OUTPUT FOR SAMPLE PERIOD 2:
THURSDAY BEFORE 5 p.m.: WITH BAG BOY

OBSERVED AND ESTIMATED VALUES AND RESIDUALS

NUMBER	OBSERVED VALUE	ESTIMATED VALUE	RESIDUAL
1	1.3170000 E 00	1.3656734 E 00	-4.8673401 E-02
2	1.1670000 E 00	1.1647000 E 00	2.3000143 E-03
3	1.1670000 E 00	1.1663903 E 00	6.0965177 E-04
4	1.2420000 E 00	1.2285819 E 00	1.3418110 E-02
5	1.3170000 E 00	1.2771697 E 00	3.9830330 E-02
6	1.1670000 E 00	1.1637345 E 00	3.2654672 E-03
7	1.2670000 E 00	1.2377079 E 00	2.9292130 E-02
8	1.0670000 E 00	1.1070423 E 00	-4.0042302 E-02

```
SUM OF RESIDUALS                                -1.1102230 E-16
SUM OF SQUARES OF RESIDUALS                      6.6133408 E-03

MQ UNDERFLOW AT       021435

MQ UNDERFLOW AT       021436
MEAN OF RESIDUALS                               -1.3877788 E-17
STANDARD DEVIATION OF RESIDUALS                  3.0736997 E-02
DURBIN-WATSON STATISTIC                          1.5551064 E 00
CORRELATION OF RESIDUALS                         0.21285   0.25525
                                                -0.47945   0.41669
NUMBER OF RUNS                                     3
      POSITIVE SIGNS                               6
      NEGATIVE SIGNS                               2
      MEAN                                       4.0000000 E 00
      STANDARD DEVIATION                         9.2582010 E-01

MQ UNDERFLOW AT       022263

MQ UNDERFLOW AT       022270

AC,MQ UNDERFLOW AT    022273

MQ UNDERFLOW AT       022312

AC,MQ UNDERFLOW AT    022315

MQ UNDERFLOW AT       022323

MQ UNDERFLOW AT       022326
COEFFICIENT OF SKEWNESS                         -5.8200396 E-01
      STANDARD DEVIATION                         7.5210144 E-01
COEFFICIENT OF KURTOSIS                         -4.9448625 E-01
      STANDARD DEVIATION                         1.4808805 E 00
```

TABLE C - 7

COMPUTER OUTPUT FOR SAMPLE PERIOD 3:
THURSDAY AFTER 5 p.m.: NO BAG BOY

REGRESSION COEFFICIENTS

VARIABLE	COEFFICIENT	STANDARD ERROR	T-VALUE	VARIANCE (PERCENT)
0	1.3146397 E 00			
1	1.8079530 E-01	3.3963391 E-02	5.32	82.53
TOTAL VARIANCE				82.53

ANALYSIS OF VARIANCE TABLE

SOURCE	D.F.	SUM OF SQUARES	MEAN SQUARE	F-RATIO
REGRESSION	1	3.4444148 E-01	3.4444148 E-01	28.34
ERROR	6	7.2931395 E-02	1.2155233 E-02	
TOTAL	7	4.1737288 E-01		

STANDARD ERROR OF ESTIMATE 1.1025077 E-01

TABLE C - 7 (Continued)

COMPUTER OUTPUT FOR SAMPLE PERIOD 3:
THURSDAY AFTER 5 p.m.: NO BAG BOY

REGRESSION COEFFICIENTS

VARIABLE	COEFFICIENT	STANDARD ERROR	T-VALUE	VARIANCE (PERCENT)
C	8.9090036 E-01			
1	1.5850932 E-01	1.0152318 E-01	1.56	82.53
2	2.5498595 E-01	1.0816921 E 00	0.24	0.19
TOTAL VARIANCE				82.72

ANALYSIS OF VARIANCE TABLE

SOURCE	D.F.	SUM OF SQUARES	MEAN SQUARE	F-RATIO
REGRESSION	2	3.4524311 E-01	1.7262155 E-01	11.97
ERROR	5	7.2129773 E-02	1.4425955 E-02	
TOTAL	7	4.1737288 E-01		

STANDARD ERROR OF ESTIMATE 1.2010809 E-01

EFFECT OF LAST VARIABLE
CONTRIBUTION TO SUM OF SQUARES 8.0162227 E-04
RATIO OF CONTRIBUTION TO ERROR MEAN SQUARE 0.06

DETERMINANT OF CORRELATION MATRIX 1.3282290 E-01

TABLE C - 7 (Continued)

COMPUTER OUTPUT FOR SAMPLE PERIOD 3:
THURSDAY AFTER 5 p.m.: NO BAG BOY

OBSERVED AND ESTIMATED VALUES AND RESIDUALS

NUMBER	OBSERVED VALUE	ESTIMATED VALUE	RESIDUAL
1	3.6170000 E 00	3.4863399 E 00	1.3066010 E-01
2	3.0850000 E 00	2.9575408 E 00	1.2745921 E-01
3	2.8350000 E 00	2.9353152 E 00	-1.0031520 E-01
4	3.1170001 E 00	3.2532836 E 00	-1.3628355 E-01
5	3.2420000 E 00	3.3336476 E 00	-9.1647599 E-02
6	3.0230000 E 00	2.9977467 E 00	2.5253359 E-02
7	3.1170001 E 00	3.0864492 E 00	3.0550862 E-02
8	2.8670000 E 00	2.8526772 E 00	1.4322822 E-02

SUM OF RESIDUALS	-1.7763568 E-15
SUM OF SQUARES OF RESIDUALS	7.2129773 E-02
MEAN OF RESIUDALS	-2.2204460 E-16
STANDARD DEVIATION OF RESIUDALS	1.0150987 E-01
DURBIN-WATSON STATISTIC	9.5847752 E-01
CORRELATION OF RESIDUALS	0.45493 -0.65756
	0.15994 -0.83079
NUMBER OF RUNS	3
POSITIVE SIGNS	5
NEGATIVE SIGNS	3
MEAN	4.7500000 E 00
STANDARD DEVIATION	1.2137604 E 00

MQ UNDERFLOW AT 022263

MQ UNDERFLOW AT 022270

AC,MQ UNDERFLOW AT 022273

MQ UNDERFLOW AT 022312

AC,MQ UNDERFLOW AT 022315

MQ UNDERFLOW AT 022326

COEFFICIENT OF SKEWNESS	6.9919911 E-03
STANDARD DEVIATION	7.5210144 E-01
COEFFICIENT OF KURTOSIS	-1.4336311 E 00
STANDARD DEVIATION	1.4808805 E 00

TABLE C - 8

COMPUTER OUTPUT FOR SAMPLE PERIOD 3:
THURSDAY AFTER 5 p.m.: WITH BAG BOY

REGRESSION COEFFICIENTS

VARIABLE	COEFFICIENT	STANDARD ERROR	T-VALUE	VARIANCE (PERCENT)
C	-7.5224385 E-01			
2	9.5950174 E-01	1.8083646 E-01	5.31	82.43
TOTAL VARIANCE				82.43

ANALYSIS OF VARIANCE TABLE

SOURCE	D.F.	SUM OF SQUARES	MEAN SQUARE	F-RATIO
REGRESSION	1	8.5458614 E-02	8.5458614 E-02	28.15
ERROR	6	1.8213256 E-02	3.0355426 E-03	
TOTAL	7	1.0367187 E-01		

STANDARD ERROR OF ESTIMATE 5.5095758 E-02

TABLE C - 8 (Continued)

COMPUTER OUTPUT FOR SAMPLE PERIOD 3:
THURSDAY AFTER 5 p.m.: WITH BAG BOY

REGRESSION COEFFICIENTS

VARIABLE	COEFFICIENT	STANDARD ERROR	T-VALUE	VARIANCE (PERCENT)
C	-1.1195436 E-01			
2	5.4064642 E-01	5.0496060 E-01	1.07	82.43
1	4.2215477 E-02	4.7393531 E-02	0.89	2.41
TOTAL VARIANCE				84.84

ANALYSIS OF VARIANCE TABLE

SOURCE	D.F.	SUM OF SQUARES	MEAN SQUARE	F-RATIO
REGRESSION	2	8.7952964 E-02	4.3976482 E-02	13.99
ERROR	5	1.5718905 E-02	3.1437809 E-03	
TOTAL	7	1.0367187 E-01		

STANDARD ERROR OF ESTIMATE 5.6069429 E-02

EFFECT OF LAST VARIABLE
CONTRIBUTION TO SUM OF SQUARES 2.4943509 E-03
RATIO OF CONTRIBUTION TO ERROR MEAN SQUARE 0.79

DETERMINANT OF CORRELATION MATRIX 1.3282290 E-01

TABLE C - 8 (Continued)

COMPUTER OUTPUT FOR SAMPLE PERIOD 3:
THURSDAY AFTER 5 p.m.: WITH BAG BOY

CBSERVEC ANC ESTIMATEC VALUES AND RESIDUALS

NUMBER	OBSERVEC VALUE	ESTIMATEC VALLE	RESIDUAL
1	1.917C000 E OC	1.8499997 E 00	6.7C00283 E-02
2	1.642C0CC E OC	1.5957089 E 00	4.6291078 E-02
3	1.567CCCC E OC	1.5779712 E 00	-1.0971212 E-02
4	1.667C000 E CC	1.7496386 E 00	-8.2638619 E-02
5	1.742C000 E CC	1.7819147 E 00	-3.9914756 E-02
6	1.642C0CC E CC	1.63C9991 E 00	1.1000842 E-02
7	1.717CCCC E OC	1.6985876 E 00	1.8412372 E-02
8	1.517C000 E OC	1.52618CC E 00	-9.1799895 E-03

```
    SUM CF RESICLALS                              -9.9920072 E-16
    SUM CF SQUARES CF RESICUALS                    1.5718905 E-02

MC  UNCERFLCW AT        021436
    MEAN CF RESIUCALS                             -1.2490009 E-16
    STANCARC CEVIATICN CF RESIUCALS                4.7387317 E-02
    CURBIN-WATSCN STATISTIC                        8.9561291 E-01
    CCRRELATICN CF RESICUALS                       0.50889 -0.58637
                                                   0.25649 -0.86162
    NUMBER CF RUNS                                   4
          PCSITIVE SIGNS                             4
          NEGATIVE SIGNS                             4
          MEAN                                     5.0000000 E 00
          STANCARC CEVIATICN                       1.3093073 E 00

   MC UNCERFLCW AT      C22263

   MC UNCERFLCW AT      C2227C

AC,MC UNCERFLCW AT      022273

   MC UNCERFLCW AT      C22312

AC,MC UNCERFLCW AT      022315

   MC UNCERFLCW AT      022323

   MC UNCERFLCW AT      C22326
      CCEFFICIENT CF SKEWNESS                     -3.9827801 E-01
            STANCARC CEVIATICN                     7.5210144 E-01
      CCEFFICIENT CF KURTCSIS                      1.3624161 E-01
            STANCARC CEVIATICN                     1.4808805 E 00
```

TABLE C - 9

COMPUTER OUTPUT FOR SAMPLE PERIOD 4:
FRIDAY: NO BAG BOY

REGRESSION COEFFICIENTS

VARIABLE	COEFFICIENT	STANDARD ERROR	T-VALUE	VARIANCE (PERCENT)
C	2.1811306 E 00			
1	6.1867112 E-02	6.9265215 E-03	8.93	93.01
TOTAL VARIANCE				93.01

ANALYSIS OF VARIANCE TABLE

SOURCE	D.F.	SUM OF SQUARES	MEAN SQUARE	F-RATIO
REGRESSION	1	6.8894134 E-02	6.8894134 E-02	79.78
ERROR	6	5.1813649 E-03	8.6356081 E-04	
TOTAL	7	7.4075498 E-02		

STANDARD ERROR OF ESTIMATE 2.9386405 E-02

TABLE C - 9 (Continued)

COMPUTER OUTPUT FOR SAMPLE PERIOD 4:
FRIDAY: NO BAG BOY

REGRESSION COEFFICIENTS

VARIABLE	COEFFICIENT	STANDARD ERROR	T-VALUE	VARIANCE (PERCENT)
C	1.0801772 E 00			
1	3.7535200 E-02	1.9969059 E-02	1.88	93.01
2	6.6931847 E-01	5.1870614 E-01	1.29	1.75
TOTAL VARIANCE				94.75

ANALYSIS OF VARIANCE TABLE

SOURCE	D.F.	SUM OF SQUARES	MEAN SQUARE	F-RATIO
REGRESSION	2	7.0188522 E-02	3.5094261 E-02	45.14
ERROR	5	3.8869762 E-03	7.7739523 E-04	
TOTAL	7	7.4075498 E-02		

STANDARD ERROR OF ESTIMATE 2.7881808 E-02

EFFECT OF LAST VARIABLE
CONTRIBUTION TO SUM OF SQUARES 1.2943887 E-03
RATIO OF CONTRIBUTION TO ERROR MEAN SQUARE 1.67

DETERMINANT OF CORRELATION MATRIX 1.0830889 E-01

TABLE C - 9 (Continued)

COMPUTER OUTPUT FOR SAMPLE PERIOD 4:
FRIDAY: NO BAG BOY

CBSERVEC ANC ESTIMATEC VALUES AND RESIDUALS

NUMBER	CBSERVEC VALUE	ESTIMATED VALUE	RESIDUAL
1	2.804C000 E CC	2.8C23973 E 00	1.6027132 E-03
2	2.523C000 E CC	2.5663796 E 00	-4.3379579 E-02
3	2.523CCCC E CC	2.4931628 E 00	2.9837194 E-02
4	2.679C000 E CC	2.6591378 E 00	1.9862214 E-02
5	2.742CCCC E CC	2.75C8647 E 00	-8.8646367 E-03
6	2.648C000 E CC	2.6452497 E 00	2.7502500 E-03
7	2.617C000 E CC	2.6356534 E 00	-1.8653353 E-02
8	2.742C000 E CC	2.7251548 E 00	1.6845197 E-02

SUM CF RESICUALS	0.
SUM CF SQUARES CF RESIDUALS	3.8869762 E-03
MEAN CF RESIUCALS	0.
STANCARC CEVIATICN OF RESILUALS	2.3564429 E-02
DURBIN-WATSCN STATISTIC	2.6143720 E 00
CCRRELATICN CF RESICUALS	-0.29957 -0.34934
	-0.46254 -0.57680
NUMBER CF RUNS	7
POSITIVE SIGNS	5
NEGATIVE SIGNS	3
MEAN	4.7500000 E 00
STANCARC CEVIATICN	1.2137604 E 00
CCEFFICIENT CF SKEWNESS	-7.2223692 E-01
STANCARC CEVIATION	7.5210144 E-01
CCEFFICIENT CF KURTOSIS	3.2299677 E-01
STANCARC CEVIATICN	1.4808805 E 00

TABLE C - 10

COMPUTER OUTPUT FOR SAMPLE PERIOD 4:
FRIDAY: WITH BAG BOY

REGRESSION COEFFICIENTS

VARIABLE	COEFFICIENT	STANDARD ERROR	T-VALUE	VARIANCE (PERCENT)
C	-1.4587735 E 00			
2	1.5628002 E 00	2.3068508 E-01	6.77	88.44
TOTAL VARIANCE				88.44

ANALYSIS OF VARIANCE TABLE

SOURCE	D.F.	SUM OF SQUARES	MEAN SQUARE	F-RATIO
REGRESSION	1	6.5154119 E-02	6.5154119 E-02	45.90
ERROR	6	8.5177567 E-03	1.4196261 E-03	
TOTAL	7	7.3671876 E-02		

STANDARD ERROR OF ESTIMATE 3.7677926 E-02

TABLE C - 10 (Continued)

COMPUTER OUTPUT FOR SAMPLE PERIOD 4:
FRIDAY: WITH BAG BOY

REGRESSION COEFFICIENTS

VARIABLE	COEFFICIENT	STANDARD ERROR	T-VALUE	VARIANCE (PERCENT)
C	-5.0784259 E-01			
2	9.7242630 E-01	7.1513832 E-01	1.36	88.44
1	2.4068894 E-02	2.7531271 E-02	0.87	1.53
TOTAL VARIANCE				89.97

ANALYSIS OF VARIANCE TABLE

SOURCE	D.F.	SUM OF SQUARES	MEAN SQUARE	F-RATIO
REGRESSION	2	6.6283496 E-02	3.3141748 E-02	22.43
ERROR	5	7.3883796 E-03	1.4776759 E-03	
TOTAL	7	7.3671876 E-02		

STANDARD ERROR OF ESTIMATE 3.8440550 E-02

EFFECT OF LAST VARIABLE
CONTRIBUTION TO SUM OF SQUARES 1.1293770 E-03
RATIO OF CONTRIBUTION TO ERROR MEAN SQUARE 0.76

DETERMINANT OF CORRELATION MATRIX 1.0830889 E-01

TABLE C - 10 (Continued)

COMPUTER OUTPUT FOR SAMPLE PERIOD 4:
FRIDAY: NO BAG BOY

CBSERVEC ANC ESTIMATEC VALUES AND RESIDUALS

NUMBER	CBSERVEC VALUE	ESTIMATEC VALUE	RESIDUAL
1	1.692COCO E CC	1.6981876 E CO	-6.1875955 E-C3
2	1.442CCCO E CC	1.4561244 E 00	-1.4124367 E-C2
3	1.417COCO E CC	1.3917916 E CO	2.5208349 E-02
4	1.567CCCC E CC	1.5503713 E 00	1.6628725 E-02
5	1.692CCCO E CC	1.6358083 E 00	5.6191663 E-02
6	1.542COCO E CC	1.5414658 E 00	5.3420991 E-04
7	1.492CCOO E CC	1.5342258 E CO	-4.2225773 E-02
8	1.567CCCC E CC	1.6C30252 E 00	-3.6025212 E-02

SUM CF RESICUALS	-0.
SUM CF SQUARES CF RESICUALS	7.3883796 E-03
MEAN CF RESIUCALS	-0.
STANCARC CEVIATICN OF RESILUALS	3.2488195 E-02
CURBIN-WATSCN STATISTIC	1.1116790 E 00
CCRRELATICN OF RESICUALS	0.12458 -0.33626
	0.37331 -0.41583
NUMBER CF RUNS	3
PCSITIVE SIGNS	4
NEGATIVE SIGNS	4
MEAN	5.0C00000 E 00
STANCARC CEVIATICN	1.3C93C73 E CO
CCEFFICIENT CF SKEWNESS	4.0518798 E-01
STANCARC CEVIATICN	7.5210144 E-01
CCEFFICIENT CF KURTOSIS	-1.7441058 E-01
STANCARC CEVIATICN	1.4808805 E 00

TABLE C - 11

COMPUTER OUTPUT FOR SAMPLE PERIOD 5:
SATURDAY: NO BAG BOY

REGRESSICN CCEFFICIENTS

VARIABLE	CCEFFICIENT	STANDARD ERRCR	T-VALUE	VARIANCE (PERCENT)
C	-8.1222C89 E CC			
2	4.226725C E CC	3.7246531 E-01	11.35	95.55
TCTAL VARIANCE				95.55

ANALYSIS OF VARIANCE TABLE

SCURCE	C.F.	SUM CF SQUARES	MEAN SQUARE	F-RATIC
REGRESSICN	1	2.6471767 E-C1	2.6471767 E-C1	128.78
ERRCR	6	1.2333827 E-C2	2.0556378 E-C3	
TCTAL	7	2.77C5149 E-C1		

STANCARC ERRCR CF ESTIMATE 4.5339142 E-02

TABLE C - 11 (Continued)

COMPUTER OUTPUT FOR SAMPLE PERIOD 5:
SATURDAY: NO BAG BOY

REGRESSION COEFFICIENTS

VARIABLE	COEFFICIENT	STANDARD ERROR	T-VALUE	VARIANCE (PERCENT)
C	-9.0752768 E 00			
2	4.7579799 E 00	1.3277117 E 00	3.58	95.55
1	-5.5503129 E-02	1.3223513 E-01	-0.42	0.15
TOTAL VARIANCE				95.70

ANALYSIS OF VARIANCE TABLE

SOURCE	D.F.	SUM OF SQUARES	MEAN SQUARE	F-RATIO
REGRESSION	2	2.6513746 E-01	1.3256873 E-01	55.64
ERROR	5	1.1914038 E-02	2.3828077 E-03	
TOTAL	7	2.7705149 E-01		

STANDARD ERROR OF ESTIMATE 4.8814011 E-02

EFFECT OF LAST VARIABLE
CONTRIBUTION TO SUM OF SQUARES 4.1978823 E-04
RATIO OF CONTRIBUTION TO ERROR MEAN SQUARE 0.18

DETERMINANT OF CORRELATION MATRIX 9.1223450 E-02

TABLE C - 11 (Continued)

COMPUTER OUTPUT FOR SAMPLE PERIOD 5:
SATURDAY: NO BAG BOY

OBSERVED AND ESTIMATED VALUES AND RESIDUALS

NUMBER	OBSERVED VALUE	ESTIMATED VALUE	RESIDUAL
1	3.2730000 E 00	3.2011398 E 00	7.1860200 E-02
2	2.8980000 E 00	2.8700600 E 00	2.7939976 E-02
3	2.9600000 E 00	2.9792564 E 00	-1.9256345 E-02
4	3.4290000 E 00	3.4382505 E 00	-9.2504507 E-03
5	3.3670000 E 00	3.3886075 E 00	-2.1607459 E-02
6	3.0850000 E 00	3.1041543 E 00	-1.9154301 E-02
7	3.2420000 E 00	3.2113678 E 00	3.0632287 E-02
8	2.9920000 E 00	3.0531639 E 00	-6.1163907 E-02

SUM OF RESIDUALS	3.5527137 E-15
SUM OF SQUARES OF RESIDUALS	1.1914038 E-02
MEAN OF RESIDUALS	4.4408921 E-16
STANDARD DEVIATION OF RESIDUALS	4.1255369 E-02
DURBIN-WATSON STATISTIC	1.2859247 E 00
CORRELATION OF RESIDUALS	0.37587 -0.00569
	-0.66424 0.35221
NUMBER OF RUNS	4
POSITIVE SIGNS	3
NEGATIVE SIGNS	5
MEAN	4.7500000 E 00
STANDARD DEVIATION	1.2137604 E 00

AC,MC UNDERFLOW AT 022273

AC,MC UNDERFLOW AT 022315

MC UNDERFLOW AT 022326

COEFFICIENT OF SKEWNESS	4.5510545 E-01
STANDARD DEVIATION	7.5210144 E-01
COEFFICIENT OF KURTOSIS	1.4769216 E-01
STANDARD DEVIATION	1.4808805 E 00

TABLE C - 12

COMPUTER OUTPUT FOR SAMPLE PERIOD 5:
SATURDAY: WITH BAG BOY

REGRESSION COEFFICIENTS

VARIABLE	COEFFICIENT	STANDARD ERROR	T-VALUE	VARIANCE (PERCENT)
C	-4.3290389 E 00			
2	2.3080817 E 00	3.3223942 E-01	6.95	88.94
TOTAL VARIANCE				88.94

ANALYSIS OF VARIANCE TABLE

SOURCE	D.F.	SUM OF SQUARES	MEAN SQUARE	F-RATIO
REGRESSION	1	7.8936395 E-02	7.8936395 E-02	48.26
ERROR	6	9.8136035 E-03	1.6356006 E-03	
TOTAL	7	8.8749998 E-02		

STANDARD ERROR OF ESTIMATE 4.0442559 E-02

TABLE C - 12 (Continued)

COMPUTER OUTPUT FOR SAMPLE PERIOD 5:
SATURDAY: WITH BAG BOY

REGRESSION COEFFICIENTS

VARIABLE	COEFFICIENT	STANDARD ERROR	T-VALUE	VARIANCE (PERCENT)
C	-6.4052314 E 00			
2	3.4652624 E 00	1.0757966 E 00	3.22	88.94
1	-1.2089704 E-01	1.0714532 E-01	-1.13	2.24
TOTAL VARIANCE				91.19

ANALYSIS OF VARIANCE TABLE

SOURCE	D.F.	SUM OF SQUARES	MEAN SQUARE	F-RATIO
REGRESSION	2	8.0928107 E-02	4.0464054 E-02	25.87
ERROR	5	7.8218909 E-03	1.5643782 E-03	
TOTAL	7	8.8749998 E-02		

STANDARD ERROR OF ESTIMATE 3.9552220 E-02

EFFECT OF LAST VARIABLE
CONTRIBUTION TO SUM OF SQUARES 1.9917126 E-03
RATIO OF CONTRIBUTION TO ERROR MEAN SQUARE 1.27

DETERMINANT OF CORRELATION MATRIX 9.1223450 E-02

TABLE C - 12 (Continued)

COMPUTER OUTPUT FOR SAMPLE PERIOD 5:
SATURDAY: WITH BAG BOY

CBSERVEC ANC ESTIMATEC VALUES ANC RESIDUALS

NUMBER	CBSERVEC VALUE	ESTIMATED VALLE	RESIDUAL
1	1.867CCOC E CC	1.8542085 E 00	1.2791530 E-02
2	1.617CCCC E CC	1.6573419 E 00	-4.0341888 E-02
3	1.742C000 E CC	1.7296275 E 00	1.2372481 E-02
4	1.992CCOC E CC	1.9705658 E 00	2.1434240 E-02
5	1.917CC00 E CC	1.9521147 E 00	-3.5114753 E-02
6	1.842CCOC E CC	1.8085203 E 00	3.3479692 E-02
7	1.842CC00 E CC	1.8833854 E 00	-4.1385457 E-02
8	1.817CCCC E CC	1.7802358 E 00	3.6764153 E-02

SUM CF RESICUALS	-2.2204460 E-15
SUM CF SQUARES CF RESICUALS	7.8218909 E-03
MEAN CF RESIUDALS	-2.7755575 E-16
STANCARC CEVIATICN OF RESIUDALS	3.3427728 E-02
CURBIN-WATSCN STATISTIC	3.2344122 E 00
CCRRELATICN CF RESICUALS	-0.76029 0.42272
	-0.82052 -0.54162
NUMBER CF RUNS	7
POSITIVE SIGNS	5
NEGATIVE SIGNS	3
MEAN	4.7500000 E 00
STANCARC CEVIATICN	1.2137604 E 00

MC UNCERFLCW AT C22263

MC UNCERFLCW AT 02227C

AC,MC UNCERFLCW AT 022273

MC UNCERFLCW AT 022312

AC,MC UNCERFLCW AT C22315

MC UNCERFLCW AT C22326

CCEFFICIENT CF SKEWNESS	-4.0175426 E-01
STANCARC CEVIATICN	7.5210144 E-01
CCEFFICIENT CF KURTOSIS	-2.0313265 E 00
STANCARC CEVIATICN	1.4808805 E 00

TABLE C - 13

COMPUTER OUTPUT FOR ALL SAMPLE PERIODS GROUPED:
NO BAG BOY

REGRESSION COEFFICIENTS

VARIABLE	COEFFICIENT	STANDARD ERROR	T-VALUE	VARIANCE (PERCENT)
0	1.7695280 E 00			
1	1.3601256 E-01	1.4762009 E-02	9.21	69.08
TOTAL VARIANCE				69.08

ANALYSIS OF VARIANCE TABLE

SOURCE	D.F.	SUM OF SQUARES	MEAN SQUARE	F-RATIO
REGRESSION	1	4.2845013 E 00	4.2845013 E 00	84.89
ERROR	38	1.9178595 E 00	5.0469987 E-02	
TOTAL	39	6.2023608 E 00		

STANDARD ERROR OF ESTIMATE 2.2465526 E-01

TABLE C - 13 (Continued)

COMPUTER OUTPUT FOR ALL SAMPLE PERIODS GROUPED:
NO BAG BOY

REGRESSICN COEFFICIENTS

VARIABLE	COEFFICIENT	STANDARD ERROR	T-VALUE	VARIANCE (PERCENT)
C	1.6695283 E CO			
1	8.7113311 E-C2	3.CC23274 E-02	2.90	69.08
2	2.2193929 E-C1	1.1979011 E-C1	1.85	2.63
TCTAL VARIANCE				71.70

ANALYSIS OF VARIANCE TABLE

SCURCE	C.F.	SUM CF SQUARES	MEAN SQUARE	F-RATIC
REGRESSICN	2	4.4473227 E CO	2.2236614 E CO	46.88
ERRCR	37	1.7550382 E CO	4.7433464 E-02	—
TCTAL	39	6.2023608 E CO		

STANCARC ERRCR CF ESTIMATE 2.1779225 E-01

EFFECT CF LAST VARIABLE
CCNTRIBUTICN TC SUM CF SQUARES 1.6282133 E-01
RATIC CF CCNTRIBUTICN TC ERROR MEAN SQUARE 3.43

DETERMINANT CF CCRRELATICN MATRIX 2.2720949 E-01

TABLE C - 13 (Continued)

COMPUTER OUTPUT FOR ALL SAMPLE PERIODS GROUPED:
NO BAG BOY

CBSERVEC ANC ESTIMATEC VALUES AND RESIDUALS

NUMBER	CBSERVEC VALUE	ESTIMATED VALUE	RESIDUAL
1	2.523C000 E CC	2.2267092 E 00	2.9629083 E-01
2	2.554CC0C E CC	2.2169605 E 00	3.3703952 E-01
3	2.117C000 E OC	2.1580958 E 00	-4.1095819 E-02
4	2.71CCC00 E CC	2.2645835 E 00	4.4541650 E-01
5	2.617C000 E CC	2.2C1591C E 00	4.1540905 E-01
6	1.96CC0CC E CC	2.1C62668 E 00	-1.4626676 E-C1
7	2.335C000 E OC	2.1821767 E 00	1.5282333 E-01
8	2.C85CCCC E CC	2.1641938 E 00	-7.9193758 E-02
9	2.648C000 E CC	2.7755836 E 00	-1.2758355 E-01
10	2.304C0C0 E CC	2.5563149 E 00	-2.5231491 E-01
11	2.335C000 E OC	2.5521978 E 00	-2.1719779 E-01
12	2.367C0CC E OC	2.6577155 E 00	-2.9071544 E-01
13	2.460C000 E CC	2.7C02896 E 00	-2.4028957 E-01
14	2.304C0CC E CC	2.5639166 E 00	-2.5991655 E-01
15	2.398C000 E CC	2.6134373 E 00	-2.1543728 E-01
16	2.179C0CC E CC	2.5C93186 E 00	-3.3031863 E-01
17	3.617C000 E CC	3.3158165 E 00	3.0118351 E-01
18	3.C85C00C E CC	3.CC55668 E 00	7.9433190 E-02
19	2.835CC00 E CC	2.9913070 E 00	-1.5630699 E-01
2C	3.117C001 E CC	3.1811076 E 00	-6.4107580 E-02
21	3.242C000 E CC	3.2271555 E C0	1.4844566 E-02
22	3.C23CC00 E OC	3.C319170 E 00	-8.9169145 E-03
23	3.117C001 E OC	3.0882737 E 00	2.8726289 E-C2
24	2.867C000 E CC	2.94C7372 E 00	-7.3737224 E-02
25	2.804C000 E CC	2.9663626 E 00	-1.6236257 E-01
26	2.523CC0C E CC	2.64C9537 E C0	-1.1795364 E-01
27	2.523CC00 E CC	2.5136352 E 00	9.3647967 E-03
28	2.679CC00 E CC	2.7710185 E 00	-9.2018466 E-02
29	2.742C000 E OC	2.9186614 E 00	-1.7666136 E-01
30	2.648C000 E CC	2.7387865 E 00	-9.0786529 E-02
31	2.617C000 E CC	2.7191777 E 00	-1.0217771 E-01
32	2.742CCCC E CC	2.8989362 E 00	-1.5693619 E-01
33	3.273C000 E OC	3.CC19534 E 00	2.7104660 E-01
34	2.898C0C0 E CC	2.9371737 E 00	-3.9173689 E-02
35	2.960C000 E CC	2.95C34C4 E 00	9.6595733 E-03
36	3.429C000 E CC	3.C758052 E 00	3.5319480 E-01
37	3.367C000 E CC	3.0537551 E C0	3.1324495 E-01
38	3.C85CC0C E CC	2.9696218 E 00	1.1537828 E-01
39	3.242CC00 E OC	2.97821C9 E 00	2.6378914 E-01
40	2.992C000 E OC	2.957376C E 00	3.4624020 E-02

TABLE C - 13 (Continued)

COMPUTER OUTPUT FOR ALL SAMPLE PERIODS GROUPED:
NO BAG BOY

SUM OF RESIDUALS	7.7715611 E-15
SUM OF SQUARES OF RESIDUALS	1.7550382 E 00
MEAN OF RESIUDALS	1.9428903 E-16
STANDARD DEVIATION OF RESIUDALS	2.1213434 E-01
DURBIN-WATSON STATISTIC	1.1301662 E 00
CORRELATION OF RESIDUALS	0.42276 0.30961
	0.38291 0.17646
NUMBER OF RUNS	17
POSITIVE SIGNS	17
NEGATIVE SIGNS	23
MEAN	2.0550000 E 01
STANDARD DEVIATION	3.0493905 E 00

MC UNDERFLOW AT C22263

MC UNDERFLOW AT C22270

AC,MC UNDERFLOW AT C22273

MC UNDERFLOW AT C22312

AC,MC UNDERFLOW AT C22315

MC UNDERFLOW AT C22326

COEFFICIENT OF SKEWNESS	6.0462700 E-01
STANDARD DEVIATION	3.7378337 E-01
COEFFICIENT OF KURTOSIS	-6.8031725 E-01
STANDARD DEVIATION	7.3260028 E-01

TABLE C - 14

COMPUTER OUTPUT FOR ALL SAMPLE PERIODS GROUPED:
WITH BAG BOY

REGRESSICN COEFFICIENTS

VARIABLE	CCEFFICIENT	STANDARD ERRCR	T-VALUE	VARIANCE (PERCENT)
C	7.9565734 E-C1			
1	9.7853357 E-C2	9.3769161 E-03	1C.44	74.13
TCTAL VARIANCE				74.13

ANALYSIS OF VARIANCE TABLE

SCURCE	C.F.	SUM CF SQUARES	MEAN SQUARE	F-RATIC
REGRESSICN	1	2.2176537 E CC	2.2176537 E CC	108.9C
ERRCR	38	7.7383C64 E-C1	2.0363964 E-02	
TCTAL	39	2.9914844 E CC		

STANCARC ERRCR CF ESTIMATE 1.4270236 E-01

TABLE C - 14 (Continued)

COMPUTER OUTPUT FOR ALL SAMPLE PERIODS GROUPED:
WITH BAG BOY

REGRESSION COEFFICIENTS

VARIABLE	COEFFICIENT	STANDARD ERROR	T-VALUE	VARIANCE (PERCENT)
C	7.2104599 E-01			
1	6.1366881 E-02	1.8732058 E-02	3.28	74.13
2	1.6559230 E-01	7.4739196 E-02	2.22	3.03
TOTAL VARIANCE				77.16

ANALYSIS OF VARIANCE TABLE

SOURCE	D.F.	SUM OF SQUARES	MEAN SQUARE	F-RATIO
REGRESSION	2	2.3082944 E 00	1.1541472 E 00	62.51
ERROR	37	6.8318995 E-01	1.8464593 E-02	
TOTAL	39	2.9914844 E 00		

STANDARD ERROR OF ESTIMATE 1.3588448 E-01

EFFECT OF LAST VARIABLE
CONTRIBUTION TO SUM OF SQUARES 9.0640690 E-02
RATIO OF CONTRIBUTION TO ERROR MEAN SQUARE 4.91

DETERMINANT OF CORRELATION MATRIX 2.2720949 E-01

TABLE C - 14 (Continued)

COMPUTER OUTPUT FOR ALL SAMPLE PERIODS GROUPED:
WITH BAG BOY

OBSERVED AND ESTIMATED VALUES AND RESIDUALS

NUMBER	OBSERVED VALUE	ESTIMATED VALUE	RESIDUAL
1	1.1670000 E 00	1.1234535 E 00	4.3546439 E-02
2	1.2420000 E 00	1.1162162 E 00	1.2578382 E-01
3	1.1420000 E 00	1.0736386 E 00	6.8361390 E-02
4	1.2420000 E 00	1.1510592 E 00	9.0940809 E-02
5	1.2420000 E 00	1.1051115 E 00	1.3688848 E-01
6	1.0670000 E 00	1.0361653 E 00	3.0834716 E-02
7	1.1670000 E 00	1.0908802 E 00	7.6119834 E-02
8	1.1170000 E 00	1.0779344 E 00	3.9065561 E-02
9	1.3170000 E 00	1.5204253 E 00	-2.0342531 E-01
10	1.1670000 E 00	1.3631378 E 00	-1.9613780 E-01
11	1.1670000 E 00	1.3602836 E 00	-1.9328363 E-01
12	1.2420000 E 00	1.4353480 E 00	-1.9334799 E-01
13	1.3170000 E 00	1.4660612 E 00	-1.4906117 E-01
14	1.1670000 E 00	1.3684467 E 00	-2.0144673 E-01
15	1.2670000 E 00	1.4044880 E 00	-1.3748798 E-01
16	1.0670000 E 00	1.3291615 E 00	-2.6216146 E-01
17	1.9170000 E 00	1.9056531 E 00	1.1346862 E-02
18	1.6420000 E 00	1.6848728 E 00	-4.2872782 E-02
19	1.5670000 E 00	1.6745961 E 00	-1.0759609 E-01
20	1.6670000 E 00	1.8100059 E 00	-1.4300586 E-01
21	1.7420000 E 00	1.8426578 E 00	-1.0065785 E-01
22	1.6420000 E 00	1.7039163 E 00	-6.1916286 E-02
23	1.7170000 E 00	1.7444776 E 00	-2.7477622 E-02
24	1.5170000 E 00	1.6383888 E 00	-1.2138884 E-01
25	1.6920000 E 00	1.6533727 E 00	3.8627278 E-02
26	1.4420000 E 00	1.4225878 E 00	1.9412191 E-02
27	1.4170000 E 00	1.3325998 E 00	8.4400199 E-02
28	1.5670000 E 00	1.5148063 E 00	5.2193672 E-02
29	1.6920000 E 00	1.6192695 E 00	7.2730508 E-02
30	1.5420000 E 00	1.4920998 E 00	4.9900153 E-02
31	1.4920000 E 00	1.4782675 E 00	1.3732497 E-02
32	1.5670000 E 00	1.6050964 E 00	-3.8096391 E-02
33	1.8670000 E 00	1.6844622 E 00	1.8253779 E-01
34	1.6170000 E 00	1.6381243 E 00	-2.1124310 E-02
35	1.7420000 E 00	1.6476217 E 00	9.4378272 E-02
36	1.9920000 E 00	1.7370248 E 00	2.5497522 E-01
37	1.9170000 E 00	1.7213709 E 00	1.9562906 E-01
38	1.8420000 E 00	1.6614636 E 00	1.8053635 E-01
39	1.8420000 E 00	1.6677270 E 00	1.7427297 E-01
40	1.8170000 E 00	1.6527259 E 00	1.6427405 E-01

TABLE C - 14 (Continued)

COMPUTER OUTPUT FOR ALL SAMPLE PERIODS GROUPED:
WITH BAG BOY

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    SUM OF RESIDUALS                            -8.8817842 E-16
    SUM OF SQUARES OF RESIDUALS                  6.8318995 E-01

 MC UNDERFLOW AT       021436
    MEAN OF RESIDUALS                           -2.2204460 E-17
    STANDARD DEVIATION OF RESIDUALS              1.3235441 E-01
    DURBIN-WATSON STATISTIC                      5.5093867 E-01
    CORRELATION OF RESIDUALS                     0.70565   0.66597
                                                 0.71958   0.32153

    NUMBER OF RUNS                                 9
          POSITIVE SIGNS                          23
          NEGATIVE SIGNS                          17
          MEAN                                   2.0550000 E 01
          STANDARD DEVIATION                     3.0493905 E 00

 MC UNDERFLOW AT       022263

 MC UNDERFLOW AT       022270

AC,MC UNDERFLOW AT     022273

 MC UNDERFLOW AT       022312

AC,MC UNDERFLOW AT     022315

 MC UNDERFLOW AT       022323

 MC UNDERFLOW AT       022326
    COEFFICIENT OF SKEWNESS                     -1.8322501 E-01
          STANDARD DEVIATION                     3.7378337 E-01
    COEFFICIENT OF KURTOSIS                     -8.5551698 E-01
          STANDARD DEVIATION                     7.3260028 E-01
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